Math 221 Worksheet 10
October 6, 2020
Section 2.8: Related Rates

1. A cube-shaped sponge is absorbing water, making it expand. Let $S(t)$ and $V(t)$ denote, respectively, the side-lengths and volume of the sponge at time t.

   (a) Find a function f such that $V(t) = f(S(t))$.

   (b) Describe the meaning of the derivatives $S'(t)$ and $V'(t)$. If we measure length in inches and time in minutes, what units do t, $S(t)$, $V(t)$, $S'(t)$, and $V'(t)$ have?

   (c) What is the relation between $S'(t)$ and $V'(t)$?

   (d) When the volume of the sponge is 8 cubic inches, it is absorbing water at a rate of 2 cubic inches per minute. At that instant, how fast is the length of its sides changing?

2. A snowball is melting, causing its surface area to decrease at a rate of 1 cm^2/min. How fast is its diameter decreasing at the moment when the diameter is 10 cm? (Recall that the surface area of a sphere of radius r is $4\pi r^2$.)

3. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is its surface area increasing at the moment when its sides have length 30 cm?

4. A hemispherical bowl of radius 10 cm contains water to a depth of d cm. Find the radius r of the surface of the water as a function of d. Suppose that water is being added to the bowl, causing d to increase at a rate of 0.1 cm/hr. How fast is r increasing when $d = 5$ cm?

5. A cylindrical swimming pool is being filled at a rate of 5 cubic feet per second. If the pool is 40 feet in diameter, how fast is the water level rising when the pool is one third full?

6. The radius of a cylinder is increasing at a rate of 3 in./min., and its height is decreasing at a rate of 5 in./min. At what rate is the volume of the cylinder changing when its radius is 10 in. and its height is 15 in.? Is the volume increasing or decreasing?

7. Assume that sand slowly poured onto a level surface will pile in the shape of a cone whose height is equal to the diameter of its base. If sand is poured at 2 cubic meters per second, how fast is the height of the pile increasing when its base is 8 meters in diameter? (Recall that the volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.)