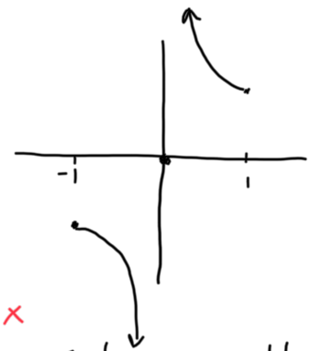
Math 221 Worksheet 11
October 8, 2020
Section 3.1: Maximum and Minimum Values

1. State the extreme value theorem. Find a function f that satisfies the following: (i) the domain of f is $[-1, 1]$, (ii) f is discontinuous at exactly one point in its domain, (iii) f attains neither a maximum nor a minimum value.

EVT: If f is continuous on $[a,b]$, then f attains both a minimum and a maximum on $[a,b]$.

$$f(x) = \begin{cases} 0, & \text{if } x = 0 \\ \frac{1}{x}, & \text{if } x \in [-1,1] \ \& \ x \neq 0. \end{cases}$$



2. Find all critical points of the function $f(x) =$ [redacted]

$x^4 + 2x^2 + 8x$

f' exists everywhere, so the critical points are the values of x such that $f'(x) = 0$.

$f'(x) = 4x^3 + 4x + 8$
$= 4(x+1)(x^2 - x + 2)$.

The only solution to $f'(x) = 0$ is $\boxed{x = -1}$

3. Find all critical points of the function $f(x) = \sin(x)\cos(x)$ in the interval $[0, 2\pi]$.

Again, f' exists everywhere, so we just need to solve $f'(x) = 0$ for $x \in [0, 2\pi]$.

$f'(x) = \cos^2(x) - \sin^2(x)$, so $f'(x) = 0$ when $\cos(x) = \pm\sin(x)$.

In $[0, 2\pi]$, this happens for $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.}$

4. Find the global minimum and maximum of the function $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$.

$f'(x) = 6x - 12$, so $x = 2$ is the only critical point.

Test at critical points and endpoints: $f(2) = -7$, $f(0) = 5$, $f(3) = -4$.

min. is -7
max. is 5

5. Find the global minimum and maximum of the function $f(x) = \sin(x) + \cos(x)$ on the interval $[0, \pi]$.

$f'(x) = \cos(x) - \sin(x)$, so the only critical point in $[0, \pi]$ is $x = \frac{\pi}{4}$.

Test at critical points and endpoints: $f(\frac{\pi}{4}) = \sqrt{2}$, $f(0) = 1$, $f(\pi) = -1$.

min. is -1
max. is $\sqrt{2}$

6. Find the global minimum and maximum of the function $f(x) = x^3 + 5x^2 - 8x + 2$ on the interval $[-1, 2]$.

7. Find the global minimum and maximum of the function $f(x) = \frac{x}{x^2+1}$ on the interval $[-2, 2]$.

8. Find all critical points of the function $f(x) = \sin(\cos(x))$. Does f have a global maximum? Why or why not?

9. Which point on the parabola defined by $y = x^2$ is closest to the point $(4, 0)$?

10. (Optional) Let P and Q be polynomials of degree 10 such that $P(0) = 0$ and $Q(0) = P'(0) = 1$. Show that the function $\frac{P}{Q}$ has at most 29 critical points.

11. (Optional) Does there exist a continuous function that has 3 local minima but only 1 local maximum?