Math 221 Worksheet 11
October 8, 2020
Section 3.1: Maximum and Minimum Values

1. State the extreme value theorem. Find a function f that satisfies the following: (i) the domain of f is $[-1, 1]$, (ii) f is discontinuous at exactly one point in its domain, (iii) f attains neither a maximum nor a minimum value.

2. Find all critical points of the function $f(x) = x^4 + 2x^2 + 8x$.

3. Find all critical points of the function $f(x) = \sin(x)\cos(x)$ in the interval $[0, 2\pi]$.

4. Find the global minimum and maximum of the function $f(x) = 3x^2 - 12x + 5$ on the interval $[0, 3]$.

5. Find the global minimum and maximum of the function $f(x) = \sin(x) + \cos(x)$ on the interval $[0, \pi]$.

6. Find the global minimum and maximum of the function $f(x) = x^3 + 5x^2 - 8x + 2$ on the interval $[-1, 2]$.

7. Find the global minimum and maximum of the function $f(x) = \frac{x}{x^2+1}$ on the interval $[-2, 2]$.

8. Find all critical points of the function $f(x) = \sin(\cos(x))$. Does f have a global maximum? Why or why not?

9. Which point on the parabola defined by $y = x^2$ is closest to the point $(3, 0)$?

10. (Optional) Let P and Q be polynomials of degree 10 such that $P(0) = 0$ and $Q(0) = P'(0) = 1$. Show that the function $\frac{P}{Q}$ has at most 29 critical points.

11. (Optional) Does there exist a continuous function that has 3 local minima but only 1 local maximum?