

Math 221 Worksheet 13
October 15, 2020
Section 3.4: Limits at Infinity and Horizontal Asymptotes

1. Evaluate the following limits (some may be ∞ or $-\infty$).

(a) $\lim_{x \rightarrow \infty} \frac{2x+1}{3x+4}$

(b) $\lim_{x \rightarrow \infty} \frac{x+3}{2x^2-10}$

(c) $\lim_{x \rightarrow -\infty} \frac{x^2+1}{10x^2-x+1}$

(d) $\lim_{x \rightarrow -\infty} \frac{3x^2+4}{x-2}$

2. The limit laws we learned also apply to limits at infinity. That being said, what is wrong with the following?

$$1 = \lim_{x \rightarrow \infty} 1 = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot x = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \lim_{x \rightarrow \infty} x = 0 \cdot \lim_{x \rightarrow \infty} x = 0$$

3. Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 + \cos(x)}{2x^2 + 4x + 1}$.

4. Evaluate $\lim_{x \rightarrow -\infty} \sqrt{9x^2 - x} + 3x$.

5. Evaluate $\lim_{x \rightarrow \infty} \frac{4x+1}{\sqrt{x^2+2}}$.

6. Evaluate $\lim_{x \rightarrow -\infty} (\sqrt[3]{x-8} - \sqrt[3]{x})$.

7. Evaluate $\lim_{x \rightarrow -\infty} \cos\left(\frac{\pi x^2 + 1}{4x^2 - 3}\right)$.

$$= \cos\left(\lim_{x \rightarrow -\infty} \frac{\pi x^2 + 1}{4x^2 - 3}\right) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{1}{\sqrt{2}}}$$

↑
because cosine
is continuous

8. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{5x^2}{x^2 - 4}$. Justify your answer.

Vertical

- $x = 2$ because $\lim_{x \rightarrow 2^+} f(x) = \infty$
- $x = -2$ because $\lim_{x \rightarrow -2^+} f(x) = -\infty$
- No others because f is continuous everywhere except $x = \pm 2$.

Horizontal

- $y = 5$ because $\lim_{x \rightarrow \infty} f(x) = 5$
- No others, because $\lim_{x \rightarrow -\infty} f(x)$ is also 5.

9. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{x^2 + x - 2}{x^2 - 1}$. Justify your answer.

Vertical

- $x = -1$ because $\lim_{x \rightarrow -1^+} f(x) = \infty$
- No others because $\lim_{x \rightarrow 1} f(x) = \frac{3}{2}$ and f is continuous everywhere except $x = \pm 1$.

Horizontal

- $y = 1$ because $\lim_{x \rightarrow \infty} f(x) = 1$.
- No others because $\lim_{x \rightarrow -\infty} f(x)$ is also 1.

10. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{x+2}{\sqrt{x^2+1}}$. Justify your answer.

Vertical

- None, because f is continuous.

Horizontal

- $y = 1$ because $\lim_{x \rightarrow \infty} f(x) = 1$.
- $y = -1$ because $\lim_{x \rightarrow -\infty} f(x) = -1$.

(• No others possible)