

Math 221 Worksheet 13
October 15, 2020
Section 3.4: Limits at Infinity and Horizontal Asymptotes

1. Evaluate the following limits (some may be ∞ or $-\infty$).

(a) $\lim_{x \rightarrow \infty} \frac{2x+1}{3x+4}$

(b) $\lim_{x \rightarrow \infty} \frac{x+3}{2x^2-10}$

(c) $\lim_{x \rightarrow -\infty} \frac{x^2+1}{10x^2-x+1}$

(d) $\lim_{x \rightarrow -\infty} \frac{3x^2+4}{x-2}$

2. The limit laws we learned also apply to limits at infinity. That being said, what is wrong with the following?

$$1 = \lim_{x \rightarrow \infty} 1 = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot x = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \lim_{x \rightarrow \infty} x = 0 \cdot \lim_{x \rightarrow \infty} x = 0$$

3. Evaluate $\lim_{x \rightarrow \infty} \frac{x^2 + \cos(x)}{2x^2 + 4x + 1}$.

4. Evaluate $\lim_{x \rightarrow -\infty} \sqrt{9x^2 - x} + 3x$.

5. Evaluate $\lim_{x \rightarrow \infty} \frac{4x+1}{\sqrt{x^2+2}}$.

6. Evaluate $\lim_{x \rightarrow -\infty} (\sqrt[3]{x-8} - \sqrt[3]{x})$.

7. Evaluate $\lim_{x \rightarrow -\infty} \cos\left(\frac{\pi x^2 + 1}{4x^2 - 3}\right)$.

8. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{5x^2}{x^2 - 4}$. Justify your answer.

9. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{x^2 + x - 2}{x^2 - 1}$. Justify your answer.

10. Find all vertical and horizontal asymptotes of the function $f(x) = \frac{x+2}{\sqrt{x^2+1}}$. Justify your answer.