Math 221 Worksheet 16
October 27, 2020
Sections 3.7: Optimization; Section 3.9: Antiderivatives

1. Show that, of all the rectangles with a given perimeter, the one with the greatest area is a square.

2. Suppose you're building a 300 foot fence to enclose a rectangular plot of land. A building adjoins one side of the plot, while the fence should make up the other three sides. What's the largest amount land that you can enclose?

3. For each of the following functions, find all antiderivatives:

(a) $f(x) = 4x + 3$

(b) $f(x) = 4\sin(x) + \sec^2(x)$

(c) $f(x) = \sqrt{x} - 2x^{-3} + (x-3)^2$.

(d) $f(x) = \sin(x)\cos(x)$

4. Use the given information to determine the function f:

(a) $f'(x) = 3x^2 + \sin(2x)$ and $f(0) = 5/2$

(b) $f''(x) = 35x^{3/2} - 9\sin(3x)$ and $f'(0) = 11$, $f(0) = -5$.

(c) $f'''(x) = \cos(x)$ and $f(0) = 1$, $f'(0) = 2$, $f''(0) = -3$.

$f''(x) = \sin(x) + C$ for some C.

$-3 = f''(0) = C$.

So $f'(x) = -\cos(X) - 3x + C'$ for some C'.

$2 = f'(0) = -1 + C'$, so $C' = 3$.

So $f(x) = -\sin(X) - \frac{3}{2}x^2 + 3x + C''$ for some C''.

$1 = f(0) = C''$.

So $f(X) = -\sin(x) - \frac{3}{2}x^2 + 3x + 1$.

5. A squirrel climbs a thin vertical tree trunk. Suppose that, when t seconds have passed, the squirrel's velocity is $v(t) = t^3 - 12t^2 + 35t$ feet per second. What is the squirrel's displacement after 8 seconds?

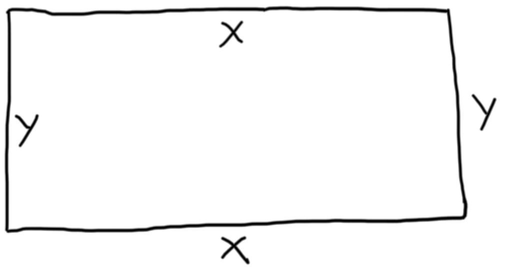
Displacement is the change in position, and position is an antiderivative of velocity:

$$s(t) = \frac{1}{4}t^4 - 4t^3 + \frac{35}{2}t^2 + C \text{ for some } C.$$

So

displacement $= s(8) - s(0) = 96$.

6. Suppose you have \$1000 to spend on fencing a rectangular plot of land with sides parallel to the cardinal directions. If the east and west sides of the plot cost \$10 per foot to fence and the north and south sides cost \$5 per foot, what is the largest amount of land you can enclose?



The cost is $5 \cdot 2x + 10 \cdot 2y$, so we have

$$10x + 20y = 1000.$$

We want to maximize the area xy. This can be written as $A(x) = x \cdot \frac{1000 - 10x}{20} = x\left(50 - \frac{x}{2}\right)$.

Critical points: $A'(x) = 50 - x = 0 \iff x = 50$.

Note that $0 \le x \le 100$ (due to the cost constraint), so $A(x)$ is maximized at either $x = 50, 0$, or 100.

The maximum occurs at $x = 50$ and the value is 1250 square feet.

7. Which point on the parabola defined by $y = x^2$ is closest to the point $(3, 0)$?