Math 221 Worksheet 17
October 29, 2020
Section 4.1: Area and Riemann Sums; Section 3.9: Antiderivatives

1. Evaluate the following sums:

   (a) $\sum_{i=0}^{3} 2^i$

   (b) $\sum_{n=1}^{4} n^2$

   (c) $\sum_{j=10}^{100} (-1)^j$

2. (a) Estimate the area under the graph of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 5$ by forming a Riemann sum of four rectangles using the right endpoints. Is your estimate too high or too low?

(b) Repeat part (a) using the left endpoints.

3. (a) Write the area under the graph of $f(x) = x^3$ from $x = 0$ to $x = 3$ as a limit of Riemann sums. (You do not need to evaluate the limit.)

(b) Write the area under the graph of $f(x) = \frac{2x}{x^2+1}$ from $x = 1$ to $x = 3$ as a limit of Riemann sums. (You do not need to evaluate the limit.)

4. (a) Using the fact that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{(2n+1)(n+1)n}{6}$, evaluate $\lim_{n\to\infty} \sum_{j=1}^{n} \left(j \cdot \frac{2}{n}\right)^2 \frac{2}{n}$.

(b) Explain why the limit from part (a) is equal to the area under the graph of $f(x) = x^2$ from $x = 0$ to $x = 2$.

5. Use the idea of area to evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \sqrt{1 - \left(\frac{j}{n}\right)^2}$. (Hint: Think about the function $f(x) = \sqrt{1 - x^2}$.)

6. A stone is dropped off a cliff and hits the ground at a speed of 120 feet per second. Assuming the acceleration due to gravity is 32 feet per second, what is the height of the cliff?