1. Find the area under the graph of $f(x) = x^2$ from x = 0 to x = 3 by evaluating a limit of Riemann sums.

The area is
$$\lim_{n \to \infty} \sum_{i=1}^{n} (i \cdot \frac{3}{n})^2 \cdot \frac{3}{n}$$
. (Here, right endpoints
were used, but other "sample points" would give the
same limit.)
 $27 \quad \frac{7}{n} \cdot 2 \quad (recall \div |^2 + 2^2 + \dots + n^2 = \frac{(2n+1)(n+1)n}{6})$

So area =
$$\lim_{n \to \infty} \frac{27}{n^3} \sum_{i=1}^{2} i^2 (recall : |^2 + 2 + \dots + n) = -6 /$$

= $\lim_{n \to \infty} \frac{27}{n^3} \cdot \frac{(2n+1)(n+1)n}{6}$
= $\frac{54}{6}$
= 9.

2. Evaluate the following definite integrals:



3. Suppose f and g are continuous functions on [0, 4] satisfying $\int_0^1 f(x)dx = 4$, $\int_0^4 f(x)dx = -6$, $\int_0^1 g(x)dx = -2$, and $\int_1^4 g(x)dx = 13$. Determine the following:

(a)
$$\int_{1}^{4} f(x) dx + \int_{1}^{1} g(x) dx$$

= $\int_{0}^{4} f(x) dx - \int_{0}^{1} f(x) dx + 0 = -6 - 4 = -10.$

(b)
$$\int_{0}^{4} f(x) - g(x)dx$$

 $= \int_{0}^{4} f(x) dx - \int_{0}^{4} g(x) dx = \int_{0}^{4} f(x) dx - \left(\int_{0}^{1} g(x) dx + \int_{1}^{4} g(x) dx\right)$
 $= -6 - (-2 + 13) = -17.$
(c) $\int_{1}^{1} 2f(x) + 3g(x)dx$
 $= -\int_{1}^{4} (2f(x) + 3g(x)) dx = -2\int_{1}^{4} f(x) dx - 3\int_{1}^{4} g(x) dx$
 $= -2\left(\int_{0}^{4} f(x) dx - \int_{0}^{1} f(x) dx\right) - 3\int_{1}^{4} g(x) dx$
Explain why $2 \le \int_{-1}^{1} \sqrt{1 + x^{2}} dx \le 2\sqrt{2}.$
 $= -2\left(-6 - 4\right) - 3 \cdot 13 = -19.$
 $1 \le \sqrt{1 + x^{2}} \le \sqrt{2}$ for all $x \in [-1, 1]$, so
 $1 \le \sqrt{1 + x^{2}} \le \sqrt{2}$ for all $x \le \sqrt{2}(1 - (-1)).$

- 5. Estimate the following definite integrals. (Hint: first determine the maximum and minimum values of the integrand on the interval over which you're integrating.)
 - (a) $\int_{1}^{3} x^{2} dx$ $| \leq \chi^{2} \leq 9$ for all $\chi \in [1, 3]$, so $| \cdot 2 \leq \int_{1}^{3} \chi^{2} d\chi \leq 9 \cdot 2$.

4.

(b)
$$\int_{4}^{9} (\sqrt{x} + x) dx$$

 $6 \leq \sqrt{x} + x \leq |Z \text{ for all } x \in [4, 9], so$
 $6 \cdot 5 \leq \int_{4}^{9} (\sqrt{x} + x) dx \leq |Z \cdot 5.$

(c) $\int_{32}^{64} \log_2(x) dx$

$$5 \leq \log_2(x) \leq 6$$
 for all $x \in [32, 64]$, so
 $5 \cdot 32 \leq \int_{32}^{64} \log_2(x) dx \leq 6 \cdot 32$.

(d)
$$\int_{0}^{1/2} \frac{1}{\sin(\pi x) + 4} dx$$

 $\frac{1}{5} \leq \frac{1}{\sin(\pi x) + 4} \leq \frac{1}{4}$ for all $x \in [0, \frac{1}{2}]$, so

$$\frac{1}{5} \frac{1}{2} \leq \int_{0}^{\frac{1}{2}} \frac{1}{\sin(\pi^{\times}) + 4} dx \leq \frac{1}{4} \frac{1}{2}.$$

6. Let $f(x) = 1 + \sqrt{9 - x^2}$.

(a) Sketch the graph of f on the interval [-3, 0]. What is the area under the graph on this interval?



7. Let f(x) = 2x.

(a) Sketch the graph of f, and label a point z on the positive x-axis.



(b) Let F(z) be the area under the graph of f on the interval [0, z]. Determine F(z).



(c) How does F(x) relate to f(x)?

$$F'(x) = f(x).$$

(d) Use area to determine
$$\int_{-2}^{1} f(x) dx$$
.

$$\frac{-2}{\frac{1}{2}} \int_{-2}^{1} f(x) dx = \frac{1}{2} \cdot 1 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 4 = -3.$$

(e) Calculate F(1) - F(-2), and compare this to the integral from part (d). What's going on

$$F(1) - F(-2) = |^{2} - (-2)^{2} = -3.$$

Seems like $\int_{a}^{b} f(x) dx = F(b) - F(a)$
when $F'(x) = f(x).$