

Math 221 Worksheet 18
November 3, 2020
Section 4.2: The Definite Integral

1. Find the area under the graph of $f(x) = x^2$ from $x = 0$ to $x = 3$ by evaluating a limit of Riemann sums.

2. Evaluate the following definite integrals:

(a) $\int_{-2}^5 (3x + 2)dx$

(b) $\int_{-1}^1 \sqrt{1 - x^2}dx.$

(c) $\int_{-1}^2 |x|dx$

3. Suppose f and g are continuous functions on $[0, 4]$ satisfying $\int_0^1 f(x)dx = 4$, $\int_0^4 f(x)dx = -6$, $\int_0^1 g(x)dx = -2$, and $\int_1^4 g(x)dx = 13$. Determine the following:

(a) $\int_1^4 f(x)dx + \int_1^4 g(x)dx$

(b) $\int_0^4 (f(x) - g(x))dx$

(c) $\int_4^1 (2f(x) + 3g(x))dx$

4. Explain why $2 \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq 2\sqrt{2}$.

5. Estimate the following definite integrals. (Hint: first determine the maximum and minimum values of the integrand on the interval over which you're integrating.)

(a) $\int_1^3 x^2 dx$

(b) $\int_4^9 (\sqrt{x} + x) dx$

(c) $\int_{32}^{64} \log_2(x) dx$

(d) $\int_0^{1/2} \frac{1}{\sin(\pi x)+4} dx$

6. Let $f(x) = 1 + \sqrt{9 - x^2}$.

(a) Sketch the graph of f on the interval $[-3, 0]$. What is the area under the graph on this interval?

(b) What is $\int_{-3}^0 (1 + \sqrt{9 - x^2}) dx$?

7. Let $f(x) = 2x$.

(a) Sketch the graph of f , and label a point z on the positive x -axis.

(b) Let $F(z)$ be the area under the graph of f on the interval $[0, z]$. Determine $F(z)$.

(c) How does $F(x)$ relate to $f(x)$?

(d) Use area to determine $\int_{-2}^1 f(x)dx$.

(e) Calculate $F(1) - F(-2)$, and compare this to the integral from part (d). What's going on