Math 221 Worksheet 19
November 5, 2020
Section 4.3: The Fundamental Theorem of Calculus

1. State the fundamental theorem of calculus.

2. Use the fundamental theorem of calculus to evaluate $\int_0^3 x^2 dx$. Compare this to Problem 1 from Worksheet 18.

3. Use the fundamental theorem of calculus to determine the following:

   (a) $\frac{d}{dx}\left(\int_0^x \sqrt{1-t^2}dt\right)$

   (b) $\frac{d}{dx}\left(\int_x^{-5} (t^3-2t^2+1)dt\right)$

   (c) $\frac{d}{dx}\left(\int_2^{7x+3} t^2 dt\right)$

   (d) $\frac{d}{dx}\left(\int_2^{1/x} \arctan(t)dt\right)$

4. Let $F(x) = \int_2^x \frac{1}{1+t+t^2}dt$. Determine the region on which F is concave up.

5. Use the fundamental theorem of calculus to evaluate the following:

(a) $\int_1^4 (2x^4 - 3x^2)dx$

(b) $\int_0^4 x\sqrt{x^3}dx$

(c) $\int_0^{\frac{\pi}{4}} \sin(x)dx$

(d) $\int_0^1 (x^3 - 1)^2 dx$

6. Compute $\int_{-1}^1 (x + x^3)dx$. Given that you integrated an *odd* function, is there a geometric explanation for your answer?

7. Let f be a continuous function satisfying $\int_1^5 f(t)dt = 8$.

(a) Let $F(x) = \int_0^x f(t)dt$. Show that $\frac{F(5)-F(1)}{5-1} = 2$.

(b) Prove that there exists $x \in (1,5)$ such that $f(x) = 2$.

8. Let $f(x) = \frac{1}{3}x$ and $g(x) = \sqrt{x}$.

(a) Find all points at which the graphs of f and g intersect.

(b) Find the area of the bounded region enclosed by the graphs of f and g.

9. (**Fun/optional**) Let f be a continuous function and let c be a real number. Prove that

$$\lim_{r \to 0^+} \frac{1}{2r} \int_{c-r}^{c+r} f(x)dx = f(c).$$