Math 221 Worksheet 20
November 10, 2020
Section 4.4: Indefinite Integrals and Net Change Theorem; Section 4.5: Substitution Rule

1. Determine the following indefinite integrals:

   (a) $\int x^{3/2}\,dx$

   (b) $\int \cos(x+3)\,dx$

   (c) $\int 2x\cos(x^2)\,dx$

   (d) $\int 5x\sqrt{x^2+1}\,dx$

2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $t \in [0, 50]$ is the number of minutes since the water began flowing. Find the amount of water that flows out of the tank during the first ten minutes.

3. A particle is moving with an acceleration of $a(t) = 2t + 5$ meters per second squared at time t. The initial velocity of the particle is $v(0) = 4$. Find the velocity $v(t)$ at time t, as well as the total distance traveled over the first 10 seconds.

4. Evaluate the following definite integrals:

(a) $\int_{-\pi/4}^{0} \sin(2x)\,dx$

Let $u = 2x$. Then $du = 2\,dx$. So

$$\int_{-\frac{\pi}{4}}^{0} \sin(2x)\,dx = \int_{-\frac{\pi}{2}}^{0} \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u)\Big|_{-\frac{\pi}{2}}^{0}$$

$$= \boxed{-\frac{1}{2}}$$

(b) $\int_{0}^{\sqrt{\pi}/2} 2x\cos(x^2)\,dx$

Let $u = x^2$. Then $du = 2x\,dx$. So

$$\int_{0}^{\sqrt{\pi}/2} 2x\cos(x^2)\,dx = \int_{0}^{\pi/4} \cos(u)\,du = \sin(u)\Big|_{0}^{\pi/4} = \boxed{\frac{1}{\sqrt{2}}}$$

(c) $\int_{-1}^{1} 5x\sqrt{1-x^2}\,dx$

By symmetry: $5x\sqrt{1-x^2}$ is an <u>odd</u> function, and we're integrating over $[-1,1]$, so the integral is $\boxed{0.}$

By substitution: $u = 1 - x^2 \rightarrow du = -2x\,dx$

$$\int_{-1}^{1} 5x\sqrt{1-x^2}\,dx = \int_{0}^{0} -\frac{5}{2}\sqrt{u}\,du = \boxed{0}$$

(d) $\int_{0}^{3\pi/4} \sin(x)\cos(x)$

Let $u = \sin(x)$. Then $du = \cos(x)\,dx$. So

$$\int_{0}^{3\pi/4} \sin(x)\cos(x)\,dx = \int_{0}^{\frac{1}{\sqrt{2}}} u\,du = \frac{1}{2}u^2\Big|_{0}^{\frac{1}{\sqrt{2}}} = \boxed{\frac{1}{4}}$$

(e) $\int_{\pi/6}^{\pi/3} \frac{\sec^2(x)}{\sqrt{\tan(x)}}\,dx$

Let $u = \tan(x)$. Then $du = \sec^2(x)\,dx$. So

$$\int_{\pi/6}^{\pi/3} \frac{\sec^2(x)}{\sqrt{\tan(x)}}\,dx = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{\sqrt{u}}\,du = 2\sqrt{u}\Big|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= \boxed{2\left(3^{\frac{1}{4}} - 3^{-\frac{1}{4}}\right)}$$

5. Evaluate $\int_{-100}^{100} \left[\cos(x)^{101} \sin(x)^{101} + \sqrt[101]{\tan\left(\frac{x}{100}\right)} \right] dx$ and explain your answer. (Hint: use symmetry)

6. Let f be a continuous function satisfying $\int_0^1 f(x)\,dx = 3$. Prove that there exists an $x \in (0,1)$ such that $f(x) = 3$.