Math 221 Worksheet 20
November 10, 2020
Section 4.4: Indefinite Integrals and Net Change Theorem; Section 4.5: Substitution Rule

1. Determine the following indefinite integrals:

   (a) $\int x^{3/2}\,dx$

   (b) $\int \cos(x+3)\,dx$

   (c) $\int 2x\cos(x^2)\,dx$

   (d) $\int 5x\sqrt{x^2+1}\,dx$

2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $t \in [0, 50]$ is the number of minutes since the water began flowing. Find the amount of water that flows out of the tank during the first ten minutes.

3. A particle is moving with an acceleration of $a(t) = 2t + 5$ meters per second squared at time t. The initial velocity of the particle is $v(0) = 4$. Find the velocity $v(t)$ at time t, as well as the total distance traveled over the first 10 seconds.

4. Evaluate the following definite integrals:

(a) $\displaystyle\int_{-\pi/4}^{0} \sin(2x)\,dx$

(b) $\displaystyle\int_{0}^{\sqrt{\pi}/2} 2x\cos(x^2)\,dx$

(c) $\displaystyle\int_{-1}^{1} 5x\sqrt{1-x^2}\,dx$

(d) $\displaystyle\int_{0}^{3\pi/4} \sin(x)\cos(x)\,dx$

(e) $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sec^2(x)}{\sqrt{\tan(x)}}\,dx$

5. Evaluate $\int_{-100}^{100} \left[\cos(x)^{101} \sin(x)^{101} + \sqrt[101]{\tan\left(\frac{x}{100}\right)} \right] dx$ and explain your answer. (Hint: use symmetry)

6. Let f be a continuous function satisfying $\int_0^1 f(x)\, dx = 3$. Prove that there exists an $x \in [0, 1]$ such that $f(x) = 3$.