1. Compute the derivatives of the following functions:

(a) 
$$f(x) = \frac{e^{4x}}{5x}$$
  
 $f'(x) = \frac{5 \times e^{4x} - e^{4x}}{25 \times^{2}}$ 

- (b)  $f(x) = e^{x^2 + 5x}$  $f'(x) = e^{x^2 + 5x} (2x + 5)$
- (c)  $f(x) = e^{2x} \sin(x)$  $f'(x) = e^{2x} \cdot 2 \cdot \sin(x) + e^{2x} \cos(x)$
- (d)  $f(x) = \sin(e^{2x})$  $f'(x) = \cos(e^{2x})e^{2x} \cdot 2$

(e) 
$$F(x) = \int_{2}^{x} e^{\cos(\tan(t^{2}))} dt.$$
  
 $F'(x) = e^{\cos(\tan(x^{2}))}$  (fundamental theorem of calculus)

2. Evaluate the following definite integrals:

(a) 
$$\int_{-1}^{2} (x^{5} + e^{x}) dx$$
  

$$= \left(\frac{1}{6} \times^{6} + e^{\times}\right) \Big|_{-1}^{2}$$

$$= \frac{1}{6} \cdot 2^{6} + e^{2} - \left(\frac{1}{6}(-1)^{6} + e^{-1}\right)$$

(b) 
$$\int_{0}^{1/2} (e^{y} + 2\cos(\pi y)) dy$$
  
=  $\left( e^{y} + \frac{2}{\pi} \sin(\pi y) \right) \Big|_{0}^{\frac{1}{2}}$   
=  $e^{\frac{1}{2}} + \frac{2}{\pi} \sin(\frac{\pi}{2}) - \left( e^{0} + \frac{2}{\pi} \sin(0) \right)$   
=  $e^{\frac{1}{2}} + \frac{2}{\pi} - 1$ 

(c) 
$$\int_{1}^{2} \frac{e^{\sqrt{2x}}}{\sqrt{x}} dx$$
Let  $u = \sqrt{2x}$ . Then  $du = \frac{1}{\sqrt{2x}} dx$ . So
$$\int_{1}^{2} \frac{e^{\sqrt{2x}}}{\sqrt{x}} dx = \int_{\sqrt{2}}^{2} e^{u} \sqrt{2} du = e^{u} \sqrt{2} \int_{\sqrt{2}}^{2}$$

$$= \sqrt{2} \left( e^{2} - e^{\sqrt{2}} \right).$$
(d) 
$$\int_{0}^{\pi} \frac{\cos(x)}{e^{\sin^{2}(x)}} dx$$
Let  $u = \sin(x)$ . Then  $du = \cos(x) dx$ . So
$$\int_{0}^{\pi} \frac{\cos(x)}{e^{\sin^{2}(x)}} dx = \int_{0}^{0} \frac{1}{e^{u^{2}}} du = 0.$$
(Can also use symmetries of sine and cosine.)

3. For each of the following functions, determine its domain and range and whether it is one-to-one. If it is one-to-one, find its inverse.

(a) 
$$f(x) = 4x - 5$$
  
domain:  $\mathbb{R}$  (all real numbers)  
range:  $\mathbb{R}$   
It is one-to-one, say by the horizontal line test.  
 $f^{-1}(x) = \frac{x+5}{y}$ .  
(b)  $f(x) = x^2 - 5x$ .  
domain:  $\mathbb{R}$   
range:  $f(x) = (x - \frac{5}{2})^2 - \frac{25}{4}$  and range is  $\left[-\frac{25}{4}, \infty\right]$ .  
Not one-to-one; for example  $f(0) = f(5)$ .  
(c)  $f(x) = \sin(2x)$ .  
domain:  $\mathbb{R}$   
range:  $[-1,1]$   
Not one-to-one; for example  $f(0) = f(\pi)$ .  
(d)  $f(x) = \frac{3x-1}{2x+1}$   
domain:  $\mathbb{R} \setminus \{-\frac{1}{2}\}$  (all real numbers except  $-\frac{1}{2}$ )  
range:  $y = \frac{3x-1}{2x+1} \iff x = \frac{-y-1}{2y-3}$ , provided  $y \neq \frac{3}{2}$   
 $\longrightarrow$  range is  $\mathbb{R} \setminus \{\frac{3}{2}\}$   
It is one-to-one: If  $f(x_1) = f(x_2)$ , then algebra shows  
 $f^{-1}(x) = \frac{-x-1}{2x-3}$ .  
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4. Compute the following limits:

(a) 
$$\lim_{x \to \infty} \frac{e^{4x} - e^{-4x}}{2e^{4x} + e^{-4x}} = \lim_{x \to \infty} \frac{1 - e^{-8x}}{2e^{4x} + e^{-4x}} = \frac{1}{2}$$

(b) 
$$\lim_{x \to \infty} e^{-x} \sin(3x^2)$$
  
 $-e^{-x} \leq e^{-x} \sin(3x^2) \leq e^{-x}$  and  
 $\lim_{x \to \infty} e^{-x} = 0 = \lim_{x \to \infty} -e^{-x}$  so  
 $\lim_{x \to \infty} e^{-x} \sin(3x^2) = 0$  by the squeeze theorem.  
 $\lim_{x \to \infty} e^{-x} \sin(3x^2) = 0$  by the squeeze theorem.

5. If f is an invertible function and g is its inverse, then

$$g'(f(x)) = \frac{1}{f'(x)},$$

provided  $f'(x) \neq 0$ .

- (a) Use implicit differentiation to prove the above formula. Let y = f(x). Then g(y) = x. So g'(y)y' = 1, and thus  $g'(f(x)) = g'(y) = \frac{1}{y'} = \frac{1}{f'(x)}$ .
- (b) Given that the natural logarithm function  $\ln x$  is the inverse of the natural exponential function  $e^x$ , what is  $(\ln x)'$ ?

Let 
$$X = e^t$$
. Then  
 $(|n \times )' = \frac{1}{\frac{1}{dt}e^t} = \frac{1}{e^t} = \frac{1}{X}$ .

(c) Evaluate 
$$\int_{1}^{2} \frac{1}{x} dx$$
.  

$$\int_{1}^{2} \frac{1}{x} dx = \ln x \Big|_{1}^{2} = \ln 2.$$