Math 221 Worksheet 21
November 12, 2020
Section 6.1: Inverse Functions; Section 6.2: Exponential Functions and their Derivatives

1. Compute the derivatives of the following functions:

   (a) $f(x) = \dfrac{e^{4x}}{5x}$

   (b) $f(x) = e^{x^2+5x}$

   (c) $f(x) = e^{2x}\sin(x)$

   (d) $f(x) = \sin(e^{2x})$

   (e) $F(x) = \displaystyle\int_2^x e^{\cos(\tan(t^2))}dt.$

2. Evaluate the following definite integrals:

   (a) $\displaystyle\int_{-1}^{2} (x^5 + e^x)\, dx$

   (b) $\displaystyle\int_0^{1/2} (e^x + 2\cos(\pi x))\, dx$

(c) $\int_1^2 \frac{e^{\sqrt{2x}}}{\sqrt{x}}\,dx$

(d) $\int_0^\pi \frac{\cos(x)}{e^{\sin^2(x)}}\,dx$

3. For each of the following functions, determine its domain and range and whether it is one-to-one. If it is one-to-one, find its inverse.

   (a) $f(x) = 4x - 5$

   (b) $f(x) = x^2 - 5x$.

   (c) $f(x) = \sin(2x)$.

   (d) $f(x) = \dfrac{3x-1}{2x+1}$

4. Compute the following limits:

(a) $\lim_{x\to\infty} \dfrac{e^{4x} - e^{-4x}}{2e^{4x} + e^{-4x}}$

(b) $\lim_{x\to\infty} e^{-x}\sin(3x^2)$

5. If f is an invertible function and g is its inverse, then

$$g'(f(x)) = \frac{1}{f'(x)},$$

provided $f'(x) \neq 0$.

(a) Use implicit differentiation to prove the above formula.

(b) Given that the natural logarithm function $\ln x$ is the inverse of the natural exponential function e^x, what is $(\ln x)'$?

(c) Evaluate $\displaystyle\int_1^2 \frac{1}{x}\,dx$.