Math 221 Worksheet 23
November 24, 2020
Section 6.6: Inverse Trigonometric Functions

1. Determine the derivatives of the following functions:

   (a) $f(x) = \arcsin(4x^2)$

   (b) $g(s) = \arccos(s)\ln(2s)$

   (c) $y = (\arctan x)^2$

   (d) $f(x) = \arcsin(e^x)$

   (e) $y = \arctan\sqrt{\frac{1-x}{1+x}}$

   (f) $y = \arctan\left(\frac{x}{a}\right) + \ln\sqrt{\frac{x-a}{x+a}}$

2. Find the absolute maximum and absolute minimum of the function $f(x) = e^x - ex$ on the interval $0 \leq x \leq 5$.

3. Find y' if $\arctan(x^2 y) = 2x + xy$.

4. Find and equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point $(1, \pi)$.

5. Show that there is exactly one root of the equation $\ln(x) = 3 - x$ and that it lies between 1 and e.

6. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{1}{(y-1)^2 + 1} dy$

(b) $\displaystyle\int_0^{\sqrt{3}/4} \frac{1}{1+16x^2}\,dx$

(c) $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

(d) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

(e) $\displaystyle\int \frac{1}{\sqrt{1-x^2}\arcsin x}\,dx$

(f) $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$

(g) $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx$

(h) $\int \frac{x}{1+x^4}dx$

(i) $\int \frac{1}{\sqrt{a^2-x^2}}dx$ for $a > 0$

(j) $\int \frac{\sin(\arctan(x))}{2+2x^2}dx$

7. Find $\frac{dy}{dx}$ if $\arcsin(xy) + y^2 = \frac{y}{x}$.

8. Eliminate the trig functions from the following expressions:

   (a) $\tan(\arcsin x)$

   (b) $\sin(\arctan x)$

   (c) $\sin(2\arccos x)$

9. If $g(x) = x\arcsin(x/4) + \sqrt{16 - x^2}$, find the equation of the line tangent to $g(x)$ at $x = 2$.

10. Sketch the function $f(x) = \arctan(x) - x$ using the techniques you learned in Chapter 3.