Math 221 Worksheet 1 December 1, 2020

Section 6.8: L'Hôpital's Rule; Section 5.1: Areas Between Curves

1. For each of the following, evaluate the limit or show that it does not exist.

(a)
$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

(b)
$$\lim_{x \to 1} \frac{1-x}{1+\cos(x)}$$

(c)
$$\lim_{x \to 0^+} \sqrt{x} \ln x$$

(d)
$$\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$$

(e)
$$\lim_{t \to 0} \frac{e^{3t} - 1}{\sin(t)}$$

(f)
$$\lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}}$$

(g)
$$\lim_{x \to 1^+} \left[\ln(x^7 - 1) - \ln(x^5 - 1) \right]$$

(h)
$$\lim_{y \to 0} \frac{\sin y}{y + \tan y}$$

(i)
$$\lim_{x \to \infty} \left(1 + \frac{4}{x} \right)^x$$

2. Determine values of a and b such that $\lim_{x\to 0} \left(\frac{\sin(2x)}{x^3} + 2b + \frac{a}{x^2}\right) = 0$.

- 3. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$
 - (a) Find all points where the graphs of f and g intersect. Sketch the graphs.

(b) Find the area of the bounded region(s) enclosed by the graphs of f and g.

- 4. Repeat Problem 3 for the following pairs of functions:
 - (a) $f(x) = x^3$ and g(x) = x

(b)
$$f(x) = \sin(x)$$
 and $g(x) = 1 - \sin(x)$ for $0 \le x \le \pi$

(c)
$$f(x) = \sqrt{1 - x^2}$$
 and $g(x) = \frac{1 - x^2}{2}$ for $-1 \le x \le 1$