Math 221 Worksheet 1
December 1, 2020
Section 6.8: L'Hôpital's Rule; Section 5.1: Areas Between Curves

1. For each of the following, evaluate the limit or show that it does not exist.

   (a) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

   (b) $\lim_{x \to 1} \dfrac{1 - x}{1 + \cos(x)}$

   (c) $\lim_{x \to 0^+} \sqrt{x} \ln x$

   (d) $\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$

(e) $\lim_{t\to 0} \dfrac{e^{3t}-1}{\sin(t)}$

(f) $\lim_{x\to\infty} \dfrac{\ln(x)}{\sqrt{x}}$

(g) $\lim_{x\to 1^+} \left[\ln(x^7-1) - \ln(x^5-1)\right]$

(h) $\lim_{y\to 0} \dfrac{\sin y}{y+\tan y}$

(i) $\lim_{x\to\infty} \left(1+\dfrac{4}{x}\right)^x$

2. Determine values of a and b such that $\lim_{x\to 0}\left(\frac{\sin(2x)}{x^3}+2b+\frac{a}{x^2}\right)=0$.

3. Let $f(x)=x^2$ and $g(x)=\sqrt{x}$

   (a) Find all points where the graphs of f and g intersect. Sketch the graphs.

   (b) Find the area of the bounded region(s) enclosed by the graphs of f and g.

4. Repeat Problem 3 for the following pairs of functions:

   (a) $f(x)=x^3$ and $g(x)=x$

(b) $f(x) = \sin(x)$ and $g(x) = 1 - \sin(x)$ for $0 \leq x \leq \pi$

(c) $f(x) = \sqrt{1 - x^2}$ and $g(x) = \dfrac{1 - x^2}{2}$ for $-1 \leq x \leq 1$