Math 221 Worksheet 27
December 10, 2020
Section 5.3: Volumes by Cylindrical Shells; Section 5.5: Average Value of a Function

1. For each of the following solids, write down and integral representing its volume. (You can use any method.)

   (a) The solid obtained by rotating the region bounded by the curves $y = 0$, $x = 0$, and $y = 3 - 3x$ about the y-axis

   (b) The solid obtained by rotating the region bounded by the curves $y = 0$, $x = 2$, and $y = 3x - 3$ about the x-axis

   (c) The solid obtained by rotating the region from (b) about the y-axis

(d) The solid obtained by rotating the region from (b) around the line $x = -1$

(e) The solid obtained by rotating the region from (b) around the line $y = -2$

(f) The solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the line $x = 5$

(g) The solid obtained by rotating the region bounded by the curves $y = (x-1)^2 - 1$ and $y = 2x$ about the line $x = -4$

2. For each of the following functions f and intervals I, compute the average value of f on I.

   (a) $f(x) = \sin(2x)$, $I = [0, \pi/2]$

   (b) $f(x) = x^2 + 3$, $I = [-1, 1]$

   (c) $f(x) = \frac{\ln x}{x}$, $I = [1, 2]$

3. Let R_δ be the region bounded by the curves $x = \delta$, $x = 1$, $y = 0$, and $y = x^{-1/2}$, where $0 < \delta < 1$. Let S_δ be the solid obtained by rotating R_δ about the x-axis. Let Area(R_δ) denote the area of R_δ and let and Vol(S_δ) denote the volume of S_δ.

   (a) Determine Area(R_δ) and Vol(S_δ).

(b) Determine $\lim_{\delta \to 0^+} \text{Area}(R_\delta)$ and $\lim_{\delta \to 0^+} \text{Vol}(S_\delta)$.

(c) If we now allow δ to be zero, what can you say about the area of R_0 and the volume of S_0?