Math 221 - Week 1 - Worksheet 2

Topics: Section 1.5 - The Limit of a Function

1. Guess the value of the following limits:

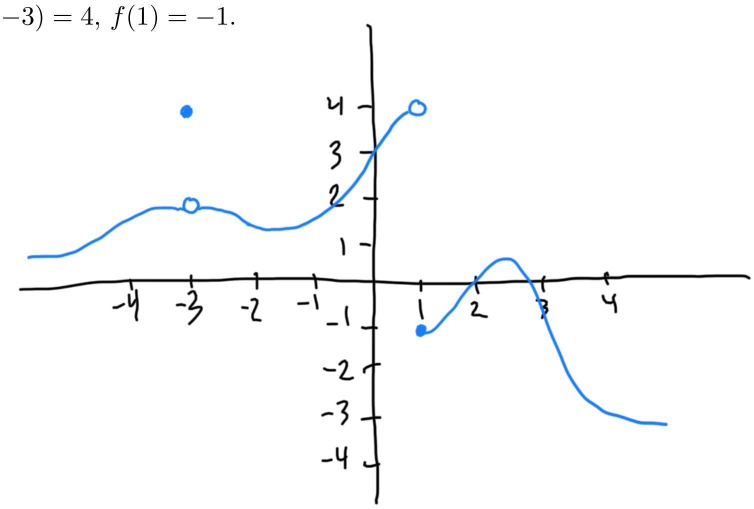
   (a) $\lim_{s\to 5} s-3$ 2, because $5-3=2$

   (b) $\lim_{u\to -2} u^2-\cos(\pi u)$ 3, because $(-2)^2-\cos(\pi(-2)) = 4-1=3$

   (c) $\lim_{v\to 4}\frac{v+3}{4v-2}$ $\frac{1}{2}$, because $\frac{4+3}{4\cdot 4-2} = \frac{7}{14} = \frac{1}{2}$

   (here it's important that the denom. is $\neq 0$)

2. Sketch the graph of a function $f$ that satisfies all of the following: $\lim_{x\to -3^-} f(x)=2$, $\lim_{x\to -3^+} f(x)=2$, $\lim_{x\to 1^-} f(x)=4$, $\lim_{x\to 1^+} f(x)=-1$, $f(-3)=4$, $f(1)=-1$.




3. Determine the following infinite limits:

   (a) $\lim_{s\to 1^-}\frac{s^2-4}{s-1}$ $+\infty$, b/c $\frac{s^2-4}{s-1} \approx \frac{-3}{\text{something small \& neg.}}$

   when s is close to 1 but less than 1

   (b) $\lim_{u\to 3^+}\frac{u^2-2u-8}{u^2-6u+9}$ $-\infty$, b/c $\frac{u^2-2u-8}{u^2-6u+9} = \frac{u^2-2u-8}{(u-3)^2}$

   $\approx \frac{-5}{\text{something small \& pos.}}$ for u close to 3

   (c) $\lim_{t\to 9^-}\frac{\sqrt{t}}{(t-9)^3}$ $-\infty$

   (similar reasoning to (b))

   (d) $\lim_{\theta\to\pi^+}\frac{\theta-4}{\sin(\theta)}$ $+\infty$ (similar reasoning to (a))

4. Consider the function $f(x) = \dfrac{2x-3}{(x-2)(x+4)}$.

(a) Find all the vertical asymptotes of $f$.

$\lim_{x\to 2^+} f(x) = \infty$, $\lim_{x\to -4^+} f(x) = \infty$ (check this),

while $f(x)$ exists everywhere except $x = 2, -4$.

So the (only) vertical asymptotes are $x=2$ and $x=-4$.

(b) Compute $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to -4^+} f(x)$, and $\lim_{x\to -4^-} f(x)$.

$\lim_{x\to 2^+} f(x) = \infty$, $\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to -4^+} f(x) = \infty$, $\lim_{x\to -4^-} f(x) = -\infty$

(c) Make a rough sketch of the function.

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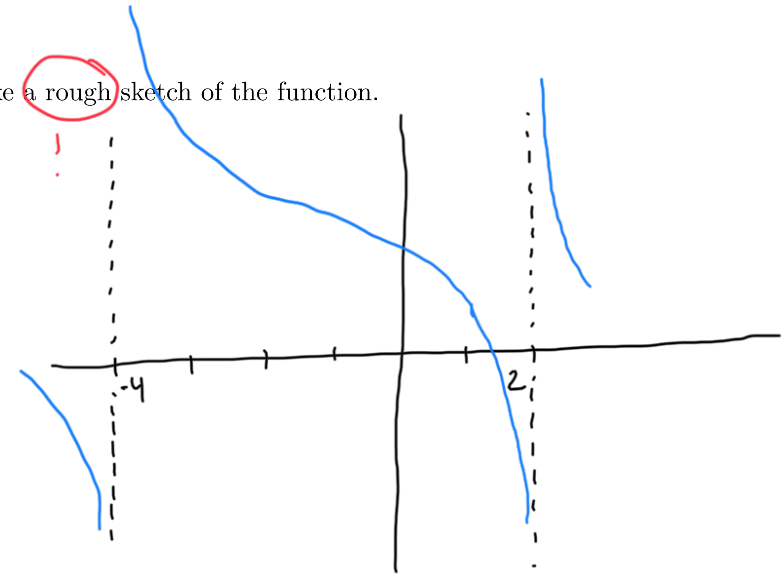


5. Consider the function $f(x) = \tan\left(\frac{1}{x}\right)$.

(a) Show that $f(x) = 0$ for $x = \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \ldots$.

If $k$ is an integer, then $\sin(k\pi) = 0$ and $\cos(k\pi) = \pm 1$.

So $\tan\left(\frac{1}{\frac{1}{k\pi}}\right) = \tan(k\pi) = \dfrac{\sin(k\pi)}{\cos(k\pi)} = 0$.

(b) Show that $f(x) = 1$ for $x = \frac{4}{\pi}, \frac{4}{5\pi}, \frac{4}{9\pi}, \ldots$.

If $k$ is an integer, then $\sin\left(\frac{\pi}{4} + k\pi\right) = \cos\left(\frac{\pi}{4} + k\pi\right)$, so

$\tan\left(\frac{1}{\frac{4}{(4k+1)\pi}}\right) = \tan\left(\frac{(4k+1)\pi}{4}\right) = \tan\left(\frac{\pi}{4} + k\pi\right) = 1$.

(c) What can you conclude about $\lim_{x\to 0^+} \tan\left(\dfrac{1}{x}\right)$?

The limit does not exist.

$\tan\left(\frac{1}{x}\right)$ "hits" both 0 and 1 infinitely many times as $x \to 0^+$.