Math 221 Worksheet 3
September 10, 2020
Topics: Section 1.6 - Calculating Limits Using the Limit Laws

1. Given that $\lim_{x \to -4} f(x) = 6$, $\lim_{x \to -4} g(x) = 0$, and $\lim_{x \to -4} h(x) = 1$, find the limits below. If the limit does not exist, explain why.

   (a) $\lim_{x \to -4} [f(x) - 4h(x)]$

   (b) $\lim_{x \to -4} \dfrac{g(x)}{3h(x)}$.

   (c) $\lim_{x \to -4} \dfrac{h(x)}{2g(x)}$.

2. Evaluate each limit and justify each step by indicating the appropriate Limit Laws.

   (a) $\lim_{a \to 2} \dfrac{a^4 - 8a + 4}{3a^2 + 16}$

   (b) $\lim_{u \to -1} \sqrt{\dfrac{2u + 5}{3u + 11}}$.

3. Evaluate the following limit, if it exists. If the limit does not exist, explain why. If you use a theorem, clearly state which theorem you are using.

   (a) $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$.

(b) $\lim_{v \to \frac{1}{2}} \frac{|2v-1|}{2v-1}$.

(c) $\lim_{x \to 0} x^4 \cos\left(\frac{1}{x}\right)$

(d) $\lim_{u \to -3} \frac{2-\sqrt{u^2-5}}{u+3}$ (Hint: multiply by the conjugate).

4. Is there a number a such that $\lim_{x \to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$ exists? If so, find the value of a and the value of the limit.

5. True or False.

(a) If $\lim_{x \to 5} f(x) = 0$ and $\lim_{x \to 5} g(x) = 0$, then $\lim_{x \to 5} \frac{f(x)}{g(x)}$ does not exist. If the answer is false, give a counterexample (that is, an example that satisfies the hypothesis but not the conclusion).

(b) If $f(x) > 1$ for all x and if $\lim_{x \to 0} f(x)$ exists, then $\lim_{x \to 0} f(x) > 1$. If the answer is false, give a counterexample.

(c) If $\lim_{x \to 6} f(x)g(x)$ exists, then the limit must be $f(6)g(6)$. If the answer is false, give a counterexample.

(d) If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$, then $\lim_{x \to 0} [f(x) - g(x)] = 0$. If the answer is false, give a counterexample.