Math 221 Worksheet 5
September 17, 2020
Section 1.8 - Continuity

1. State the definition of continuity.

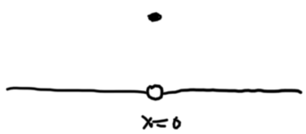
A function f is continuous at a point a if:

- f is defined at a
- $\lim_{x \to a} f(x)$ exists
- $\lim_{x \to a} f(x) = f(a)$.

2. True or False: If $\lim_{x \to 0} f(x)$ exists, then $f(x)$ is continuous at $x = 0$. (If the statement is true, explain why. If the statement is false, come up with a counterexample.)

False.

Counterexample: $f(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}$



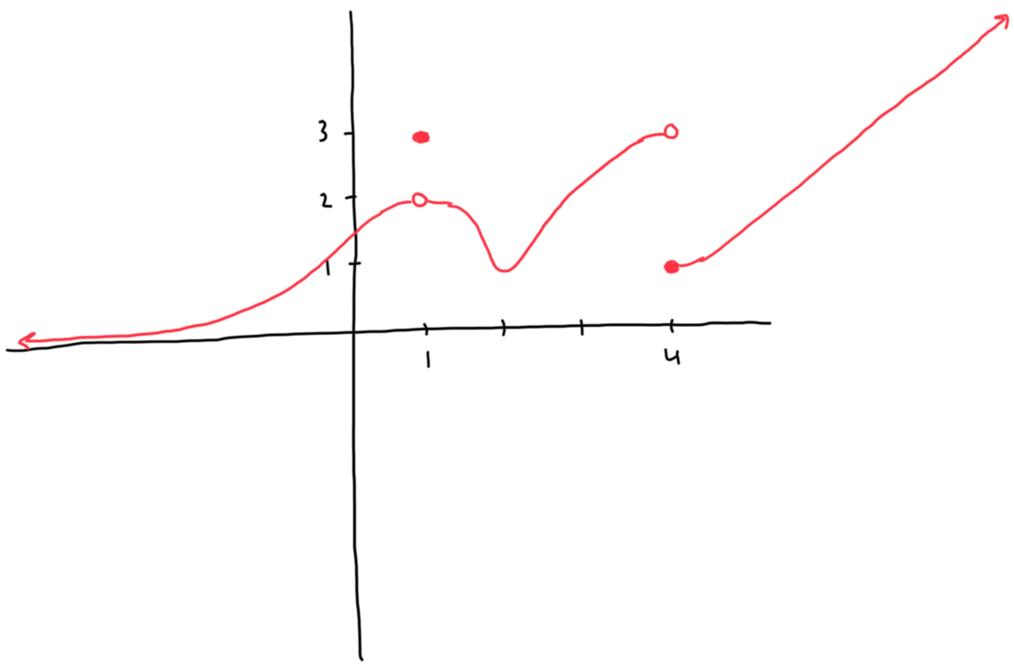
3. Draw a graph of a function $h(t)$ that satisfies all of the following properties.

(a) The domain of h is all real numbers and the range of h is all positive real numbers.

(b) $h(t)$ is not continuous at $t = 1$ and at $t = 4$.

(c) $\lim_{t \to 1^+} h(t). = 2$ and $\lim_{t \to 1^-} h(t) = 2$.

(d) $\lim_{t \to 4^+} h(t). = 1$ and $\lim_{t \to 4^-} h(t) = 3$.



4. Consider the function $g(x) = \begin{cases} x & x < -2 \\ bx^2 & x \geq -2, \end{cases}$ where b is some number.

(a) Compute $\lim_{x \to -2^-} g(x)$.

$$\lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} x = -2$$

(b) Compute $\lim_{x \to -2^+} g(x)$.

$$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} bx^2 = 4b$$

(c) Compute $g(-2)$.

$$g(-2) = b(-2)^2 = 4b$$

(d) For what value of b will $\lim_{x \to -2} g(x)$ exist?

$\lim_{x \to -2} g(x)$ exists $\Leftrightarrow \lim_{x \to -2^-} g(x) = \lim_{x \to -2^+} g(x) \Leftrightarrow -2 = 4b$

$\Leftrightarrow b = -\frac{1}{2}$

5. Let $f(x) = -8x^4 + 2x^3 - x + 1$. Use the intermediate value theorem to show that $f(c) = 0$ for some $c \in [0, 1]$.

$f(0) = 1$

$f(1) = -6$

Since f is continuous (as f is a polynomial) and 0 lies between 1 and -6, the IVT says that there exists a number c between 0 and 1 such that $f(c) = 0$.

6. Show that there exists an intersection point between the graphs of $y = \sin(x)$ and $y = 4^{x/\pi}$ in the interval $\left(\frac{-3\pi}{2}, 0\right)$.

Let $f(x) = \sin(x) - 4^{x/\pi}$. This problem is the same as showing that f has a zero in $(-\frac{3\pi}{2}, 0)$.

$f(-\frac{3\pi}{2}) = 1 - 4^{-3/2} > 0$ and $f(0) = 0 - 1 < 0$.

Since f is continuous (as a combination of common continuous functions) IVT implies the existence of $c \in (-\frac{3\pi}{2}, 0)$ such that $f(c) = 0$.

7. Locate the discontinuities of the function $f(x) = \dfrac{4}{1 + \cos(x)}$.

f is discontinuous exactly when $\cos x = -1$, which happens for $x = k\pi$ where k is an odd integer.