Math 221 Worksheet 5
September 17, 2020
Section 1.8 - Continuity

1. State the definition of continuity.

2. True or False: If $\lim_{x \to 0} f(x)$ exists, then $f(x)$ is continuous at $x = 0$. (If the statement is true, explain why. If the statement is false, come up with a counterexample.)

3. Draw a graph of a function $h(t)$ that satisfies all of the following properties.

   (a) The domain of $h$ is all real numbers and the range of $h$ is all positive real numbers.

   (b) $h(t)$ is not continuous at $t = 1$ and at $t = 4$.

   (c) $\lim_{t \to 1^+} h(t). = 2$ and $\lim_{t \to 1^-} h(t) = 2$.

   (d) $\lim_{t \to 4^+} h(t). = 1$ and $\lim_{t \to 4^-} h(t) = 3$.

4. Consider the function $g(x) = \begin{cases} x & x < -2 \\ bx^2 & x \geq -2, \end{cases}$ where $b$ is some number.

(a) Compute $\lim_{x \to -2^-} g(x)$.

(b) Compute $\lim_{x \to -2^+} g(x)$.

(c) Compute $g(-2)$.

(d) For what value of $b$ will $\lim_{x \to -2} g(x)$ exist?

5. Let $f(x) = -8x^4 + 2x^3 - x + 1$. Use the intermediate value theorem to show that $f(c) = 0$ for some $c \in [0, 1]$.

6. Show that there exists an intersection point between the graphs of $y = \sin(x)$ and $y = 4^{x/\pi}$ in the interval $\left(\frac{-3\pi}{2}, 0\right)$.

7. Locate the discontinuities of the function $f(x) = \dfrac{4}{1 + \cos(x)}$.