- 1. Let $f(x) = 2x^2$.
 - (a) Find the slope of the line through the points (a, f(a)) and (b, f(b)).

$$\frac{\text{rise}}{\text{run}} = \frac{f(b) - f(a)}{b - a} = \frac{2b^2 - 2a^2}{b - a} \quad (assuming \ a \neq b)$$

(b) Compute
$$\lim_{b \to 1} \frac{f(b) - f(1)}{b - 1}$$
.

$$\lim_{b \to 1} \frac{f(b) - f(1)}{b - 1} = \lim_{b \to 1} \frac{2b^2 - 2}{b - 1} = \lim_{b \to 1} \frac{2(b + 1)(b - 1)}{b - 1}$$

$$= \lim_{b \to 1} 2(b + 1) = 4$$

(c) Write the equation of the line tangent to the graph of f at the point (1, f(1)).

Its slope is
$$f'(1) = 4$$
 (from part(b)).
It passes through (1,2), so the equation is
 $y - 2 = 4(x - 1)$.

2. Use the definition of the derivative to find the derivative of the function $f(x) = 3x^2 + 4$ at the point x = 2. $3(2+h)^2 + 4 - (3 \cdot 2^2 + 4)$

$$f'(z) = \lim_{h \to 0} \frac{3(ch+3)(3-2+q)}{h}$$

= $\lim_{h \to 0} \frac{12h+3h^2}{h}$
= $\lim_{h \to 0} (12+3h) = 12$
h $\to 0$

3. Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x-2}$ at the point x = -1. $f'(-1) = \lim_{h \to 0} \frac{1}{-1+h-2} - \frac{1}{-1-2}$ 3 + h-3

$$= \lim_{h \to 0} \frac{1}{(h-3)\cdot 3} = \lim_{h \to 0} \frac{1}{(h-3)\cdot 3} = -\frac{1}{9}$$

4. Use the definition of the derivative to find f'(6) where $f(x) = \sqrt{x-4}$.

$$f'(6) = \lim_{h \to 0} \frac{\sqrt{6 + h - 4} - \sqrt{6 - 4}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2 + h} - \sqrt{2}}{h} \cdot \frac{\sqrt{2 + h} + \sqrt{2}}{\sqrt{2 + h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{2 + h} + \sqrt{2})} = \lim_{h \to 0} \frac{1}{\sqrt{2 + h} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$
Consider the function $f(\mathbf{x}) = \frac{3}{2 + 2}$.

5.2+x

> (a) Using the definition of the derivative, find the slope of the tangent line to the graph of f at the point (-1, f(-1)).

$$slope of tangent line at (-1, f(-1)) = f'(-1)$$

= $\lim_{h \to 0} \frac{\frac{3}{2+-1+h} - \frac{3}{2+-1}}{h}$
= $\lim_{h \to 0} \frac{3 - 3(h+1)}{h+1} = \lim_{h \to 0} \frac{-3}{h+1} = -3$

(b) Find the equation of the tangent line from part (a).

The line has sbpe -3 and passes through
$$(-1, 3)$$
,
so its equation is
 $y-3 = -3(x+1)$.

6. Suppose the position of a car at time t is given by the function $s(t) = t - t^2$.

(a) Find the average velocity of the car from t = 0 to $t = \frac{1}{2}$.

$$\frac{\text{displacement}}{\text{trave}} = \frac{s(\frac{1}{2}) - s(0)}{\frac{1}{2} - 0} = \frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

(b) Find the instantaneous velocity of the car at time t = 1.

This is

$$S'(1) = \lim_{h \to 0} \frac{1+h - (1+h)^{2} - (1-1^{2})}{h}$$

$$= \lim_{h \to 0} \frac{-h - h^{2}}{h} = \frac{1}{h} (-1 - h) = -1$$
(c) At what time is the car stopped?
Velocity is $S'(t) = \lim_{h \to 0} \frac{t+h - (t+h)^{2} - (t-t^{2})}{h}$

$$= \lim_{h \to 0} \frac{h - 2th - h^{2}}{h} = \lim_{h \to 0} (1 - 2t - h) = 1 - 2t.$$
This is 0 when $t = \frac{1}{2}$.

7. Use the definition of the derivative to find f'(x) where $f(x) = \frac{1}{\sqrt{x+1}}$.

$$f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{x+h}+1} - \frac{1}{\sqrt{x+1}} -$$

continuous at x = 0? Is it differentiable at x = 0? It is continuous at 0:

$$\lim_{X \to 0^+} f(x) = \lim_{X \to 0^+} 0 = 0 \qquad \lim_{X \to 0^+} f(x) = \lim_{X \to 0^+} x^2 = 0$$

and $f(0) = 0$.

If Ts differentiable at 0:

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0, \quad \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = 0.$$
For which values of a and b is the function
$$\lim_{h \to 0^+} \frac{h}{h} = 0.$$

10. For which values of a and b is the function

$$f(x) = \begin{cases} ax^2 + b & : x < 1 \\ x - x^2 & : x \ge 1 \end{cases}$$

differentiable at
$$x = 1$$
?
If f is differentiable at $x=1$, then it is continuous there, so $b=-a$.
 $\lim_{h \to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^-} \frac{a(1+h)^2-a}{h} = \lim_{h \to 0^-} \frac{2ah+ah^2}{h} = 2a$,
 $\lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^+} \frac{1+h-(1+h)^2}{h} = \lim_{h \to 0^+} \frac{-h-h^2}{h} = -1$.
 f is differentiable at $x=1$ if and only if these side
 $\lim_{h \to 0^+} f$ are equal, or equivalently, $a = -\frac{1}{2}$, $b = \frac{1}{2}$.