Math 221 Worksheet 6
September 22, 2020
Sections 2.1 and 2.2 - Derivatives

1. Let $f(x) = 2x^2$.

   (a) Find the slope of the line through the points $(a, f(a))$ and $(b, f(b))$.

   (b) Compute $\lim_{b \to 1} \frac{f(b) - f(1)}{b - 1}$.

   (c) Write the equation of the line tangent to the graph of f at the point $(1, f(1))$.

2. Use the definition of the derivative to find the derivative of the function $f(x) = 3x^2 + 4$ at the point $x = 2$.

3. Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x - 2}$ at the point $x = -1$.

4. Use the definition of the derivative to find $f'(6)$ where $f(x) = \sqrt{x-4}$.

$$f'(6) = \lim_{h \to 0} \frac{\sqrt{6+h-4} - \sqrt{6-4}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \cdot \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})} = \lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

5. Consider the function $f(\cancel{1}) = \dfrac{3}{2+x}$.

(a) Using the definition of the derivative, find the slope of the tangent line to the graph of f at the point $(-1, f(-1))$.

slope of tangent line at $(-1, f(-1)) = f'(-1)$

$$= \lim_{h \to 0} \frac{\frac{3}{2+-1+h} - \frac{3}{2+-1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3 - 3(h+1)}{h+1}}{h} = \lim_{h \to 0} \frac{-3}{h+1} = -3$$

(b) Find the equation of the tangent line from part (a).

The line has slope -3 and passes through $(-1, 3)$, so its equation is

$$y - 3 = -3(x+1).$$

6. Suppose the position of a car at time t is given by the function $s(t) = t - t^2$.

(a) Find the average velocity of the car from $t = 0$ to $t = \frac{1}{2}$.

$$\frac{\text{displacement}}{\text{time}} = \frac{s(\frac{1}{2}) - s(0)}{\frac{1}{2} - 0} = \frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

(b) Find the instantaneous velocity of the car at time $t = 1$.

This is

$$s'(1) = \lim_{h \to 0} \frac{1+h-(1+h)^2 - (1-1^2)}{h}$$

$$= \lim_{h \to 0} \frac{-h-h^2}{h} = \lim_{h \to 0}(-1-h) = -1$$

(c) At what time is the car stopped?

velocity is $s'(t) = \lim_{h \to 0} \dfrac{t+h-(t+h)^2 - (t-t^2)}{h}$

$$= \lim_{h \to 0} \frac{h - 2th - h^2}{h} = \lim_{h \to 0} (1 - 2t - h) = 1 - 2t.$$

This is 0 when $t = \frac{1}{2}$.

7. Use the definition of the derivative to find $f'(x)$ where $f(x) = \frac{1}{\sqrt{x+1}}$.

8. Let $f(x) = x + |x|$. What is $f'(c)$ if $c > 0$? What is $f'(c)$ if $c < 0$? What about $f'(0)$?

9. Is the function
$$f(x) = \begin{cases} 0, & x \leq 0 \\ x^2, & x > 0 \end{cases}$$
continuous at $x = 0$? Is it differentiable at $x = 0$?

10. For which values of a and b is the function
$$f(x) = \begin{cases} ax^2 + b & : x < 1 \\ x - x^2 & : x \geq 1 \end{cases}$$
differentiable at $x = 1$?