

Math 221 Worksheet 6
September 22, 2020
Sections 2.1 and 2.2 - Derivatives

1. Let $f(x) = 2x^2$.

(a) Find the slope of the line through the points $(a, f(a))$ and $(b, f(b))$.

(b) Compute $\lim_{b \rightarrow 1} \frac{f(b) - f(1)}{b - 1}$.

(c) Write the equation of the line tangent to the graph of f at the point $(1, f(1))$.

2. Use the definition of the derivative to find the derivative of the function $f(x) = 3x^2 + 4$ at the point $x = 2$.

3. Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x-2}$ at the point $x = -1$.

4. Use the definition of the derivative to find $f'(6)$ where $f(x) = \sqrt{x-4}$.

5. Consider the function $f(x) = \frac{3}{2+x}$.

(a) Using the definition of the derivative, find the slope of the tangent line to the graph of f at the point $(-1, f(-1))$.

(b) Find the equation of the tangent line from part (a).

6. Suppose the position of a car at time t is given by the function $s(t) = t - t^2$.

(a) Find the average velocity of the car from $t = 0$ to $t = \frac{1}{2}$.

(b) Find the instantaneous velocity of the car at time $t = 1$.

(c) At what time is the car stopped?

7. Use the definition of the derivative to find $f'(x)$ where $f(x) = \frac{1}{\sqrt{x+1}}$.

8. Let $f(x) = x + |x|$. What is $f'(c)$ if $c > 0$? What is $f'(c)$ if $c < 0$? What about $f'(0)$?

9. Is the function

$$f(x) = \begin{cases} 0, & x \leq 0 \\ x^2, & x > 0 \end{cases}$$

continuous at $x = 0$? Is it differentiable at $x = 0$?

10. For which values of a and b is the function

$$f(x) = \begin{cases} ax^2 + b & : x < 1 \\ x - x^2 & : x \geq 1 \end{cases}$$

differentiable at $x = 1$?