Math 221 Worksheet 8
September 29, 2020
Sections 2.4 and 2.5 - Trigonometric Functions and Chain Rule

1. Compute the derivative of each of the following functions:

   (a) $f(x) = x\sin(x)$

   $f'(x) = 1 \cdot \sin(x) + x \cdot \cos(x)$

   (b) $g(t) = \dfrac{4t^2}{\cos(t)}$

   $$g'(t) = \frac{\cos(t) \cdot 8t - 4t^2(-\sin(t))}{\cos^2(t)}$$

   (c) $f(x) = \tan(x)$

   $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, so

   $$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \sec^2(x)$$

   (d) $g(v) = v^3\sec(v)$

   $v^3\sec(v) = \dfrac{v^3}{\cos(v)}$, so

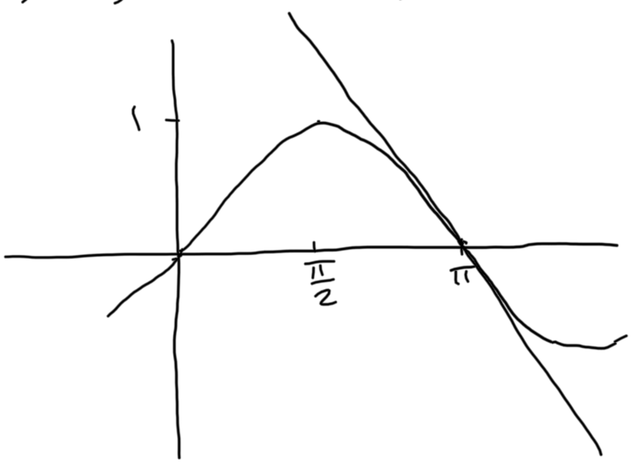
   $$g'(v) = \frac{\cos(v) \cdot 3v^2 - v^3(-\sin(v))}{\cos^2(v)}$$

2. Let $f(x) = \sin(x)$. Find the equation for the line tangent to the graph of f at the point $(\pi, f(\pi))$. Sketch the graph and tangent line.

Slope is $f'(\pi) = \cos(\pi) = -1$,

passes through $(\pi, 0)$, so the equation is

$y = -(x - \pi)$



3. Evaluate the following limits:

   (a) $\lim_{x \to 0} \frac{\sin x}{x 2^x}$

   (b) $\lim_{x \to 0} \frac{\tan x}{x}$

   (c) $\lim_{\theta \to 0} \frac{\sin(6\theta)}{3\theta}$

   (d) $\lim_{x \to 0} \frac{1 - \cos(x)}{4x \sin(x)}$

4. Recall that a function f is *even* if $f(-x) = f(x)$ for all x and *odd* if $f(-x) = -f(x)$ for all x. Show that if f is even, then f' is odd.

5. Use the chain rule to find the derivative of each of the following functions:

   (a) $f(x) = (2x + 1)^2$

   (b) $f(x) = \sin(4x)$

(c) $f(x) = \sqrt{2 + x^2} + (2 + x^2)^3$

(d) $f(x) = \sqrt{\dfrac{x-1}{x+1}}$

6. Let $g(x) = f(\frac{1}{x^2})$, where f is a differentiable function satisfying $f(3) = 5$, $f(\frac{1}{9}) = 7$, $f'(3) = 11$, and $f'(\frac{1}{9}) = 13$. Find the equation for the line tangent to the graph of g at the point $(3, g(3))$.

7. Find the 100th derivative of the function $f(x) = \cos(2x + 1)$.

8. Suppose that f is a twice-differentiable function satisfying $f(x^2) = f(x) + x^2$. What are $f'(1)$ and $f''(1)$?

9. Suppose that f is a differentiable function satisfying $f(x)^3 = x - 1 - f(x^2)$. What is $f'(1)$?