Math 221 Worksheet 8
September 29, 2020
Sections 2.4 and 2.5 - Trigonometric Functions and Chain Rule

1. Compute the derivative of each of the following functions:

   (a) $f(x) = x\sin(x)$

   (b) $g(t) = \dfrac{4t^2}{\cos(t)}$

   (c) $f(x) = \tan(x)$

   (d) $g(v) = v^3\sec(v)$

2. Let $f(x) = \sin(x)$. Find the equation for the line tangent to the graph of $f$ at the point $(\pi, f(\pi))$. Sketch the graph and tangent line.

3. Evaluate the following limits:

   (a) $\lim_{x \to 0} \dfrac{\sin x}{x 2^x}$

   (b) $\lim_{x \to 0} \dfrac{\tan x}{x}$

   (c) $\lim_{\theta \to 0} \dfrac{\sin(6\theta)}{3\theta}$

   (d) $\lim_{x \to 0} \dfrac{1 - \cos(x)}{4x \sin(x)}$

4. Recall that a function $f$ is *even* if $f(-x) = f(x)$ for all $x$ and *odd* if $f(-x) = -f(x)$ for all $x$. Show that if $f$ is even, then $f'$ is odd.

5. Use the chain rule to find the derivative of each of the following functions:

   (a) $f(x) = (2x + 1)^2$

   (b) $f(x) = \sin(4x)$

(c) $f(x) = \sqrt{2 + x^2} + (2 + x^2)^3$

(d) $f(x) = \sqrt{\dfrac{x-1}{x+1}}$

6. Let $g(x) = f(\frac{1}{x^2})$, where $f$ is a differentiable function satisfying $f(3) = 5$, $f(\frac{1}{9}) = 7$, $f'(3) = 11$, and $f'(\frac{1}{9}) = 13$. Find the equation for the line tangent to the graph of $g$ at the point $(3, g(3))$.

7. Find the 100th derivative of the function $f(x) = \cos(2x + 1)$.

8. Suppose that $f$ is a twice-differentiable function satisfying $f(x^2) = f(x) + x^2$. What are $f'(1)$ and $f''(1)$?

9. Suppose that $f$ is a differentiable function satisfying $f(x)^3 = x - 1 - f(x^2)$. What is $f'(1)$?