Math 221 Worksheet 9 October 1, 2020 Section 2.6: Implicit Differentiation

1. For each of the following equations find $\frac{dy}{dx}$:

(a)
$$x^2 + xy = y^2$$

(b) $\sqrt{xy} = \cos(x+y)$

(c) $\sin(x)\sin(y) = xy^2$

(d) $\tan(xy^2) = x$

2. The equation $\cos(x^2y) = 3xy^2 + y$ defines a curve. Find the line tangent to it at the point (0, 1).

3. Suppose that f is an invertible function, and let g be its inverse. Suppose additionally that f and g are differentiable, and let y = f(x). What is g'(y)?

- 4. For each of the following equations find $\frac{d^2y}{dx^2}$:
 - (a) $xy = x^2 + 1$

(b) $\sin(y) = xy$

5. The equation $x^2 + y^2 + xy = 1$ defines an ellipse. Among all points (x, y) on this ellipse, which one has the largest y-value and which one has the smallest?

6. The equation $y^2 = x^3 + x + 2$ defines a curve. At which point(s) does it have a vertical tangent line?

7. Let L be the line defined by 4y - 3x = 1. Find a circle of unit radius that contains the point (1, 1) and whose tangent line at (1, 1) is L.