## Math 221 Worksheet 9

October 1, 2020
Section 2.6: Implicit Differentiation

1. For each of the following equations find $\frac{d y}{d x}$ :
(a) $x^{2}+x y=y^{2}$
(b) $\sqrt{x y}=\cos (x+y)$
(c) $\sin (x) \sin (y)=x y^{2}$
(d) $\tan \left(x y^{2}\right)=x$
2. The equation $\cos \left(x^{2} y\right)=3 x y^{2}+y$ defines a curve. Find the line tangent to it at the point $(0,1)$.
3. Suppose that $f$ is an invertible function, and let $g$ be its inverse. Suppose additionally that $f$ and $g$ are differentiable, and let $y=f(x)$. What is $g^{\prime}(y)$ ?
4. For each of the following equations find $\frac{d^{2} y}{d x^{2}}$ :
(a) $x y=x^{2}+1$
(b) $\sin (y)=x y$
5. The equation $x^{2}+y^{2}+x y=1$ defines an ellipse. Among all points $(x, y)$ on this ellipse, which one has the largest $y$-value and which one has the smallest?
6. The equation $y^{2}=x^{3}+x+2$ defines a curve. At which point(s) does it have a vertical tangent line?
7. Let $L$ be the line defined by $4 y-3 x=1$. Find a circle of unit radius that contains the point $(1,1)$ and whose tangent line at $(1,1)$ is $L$.
