Topics: Section 5.3 Volumes by Cylindrical Shells, Section 5.5 - Average Value of a Function, and Final Review

1. Use any method to write down integrals that represent the volume of the following solids.
   (a) The solid obtained by rotating the region bounded by the x and y axes and the graph of $y = 3 - 3x$ about the y-axis.
   (b) Let T be the triangle enclosed by $1 \leq x \leq 2$ and $0 \leq y \leq 3x - 3$.
      i. The solid obtained by rotating T around the x-axis.
      ii. The solid obtained by rotating T around the y-axis.
      iii. The solid obtained by rotating T around the line $x = -1$.
      iv. The solid obtained by rotating T around the line $y = -2$.
   (c) The solid obtained by rotating the region enclosed by $y = x$ and $y = \sqrt{x}$ about the line $x = 5$.
   (d) The solid obtained by rotating the region enclosed by $y = -(x^2 - 2x)$ and the x-axis about the line $x = 3$.
   (e) The solid obtained by rotating the region enclosed by $x = 2 - y^2$, $x = y^4$; about the y-axis.
2. Compute the average value of the following functions on the given interval.
   (a) The function $f(x) = \sin(2x)$ on the interval $[0, \pi/2]$.
   (b) The function $f(x) = x^2 + 3$ on the interval $[-1, 1]$.

Final Exam Review

3. Consider the curve $2yx + 3x^2y = \sin(xy)$. Find $\frac{dy}{dx}$.
4. Find the absolute max and the absolute min of the function $f(x) = x^3 - 2x$ on the interval $[0, 4]$.
5. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep? (Recall that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.
6. Find a parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2 respectively.
7. Compute the following limits, if they exist. If the limit does not exist, decide whether it is ∞, $-\infty$ or neither.
   (a) $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|}$.
   (b) $\lim_{x \to 0} \cos\left(\dfrac{2}{x}\right) x^4$.
   (c) $\lim_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$.
8. Does the function $f(x) = \dfrac{x^3 - x^2 - 2x}{x - 2}$ have any discontinuities? If so, determine whether the discontinuity is a removable discontinuity, a jump discontinuity or an infinite discontinuity.
9. Show that the equation $3x + 2\cos(x) + 5 = 0$ has exactly one real root.
10. Suppose that f is continuous on $[0, 4]$, $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all x in $(0, 4)$. Show that $9 \leq f(4) \leq 21$.
11. Find the point on the ellipse $\frac{x^2}{9} + y^2 = 1$ that is closest to the point $(2, 0)$.
12. Find f if $f''(x) = 5x^3 + 6x^2 + 2$, with $f(0) = 3$ and $f(1) = -2$.
13. Find the area of the region bounded by the curves $y = e^x$, $y = e^{-x}$, $x = -2$ and $x = 1$.
14. Write an integral that represents the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 2$.