Math 221 Sec 003 Quiz 8
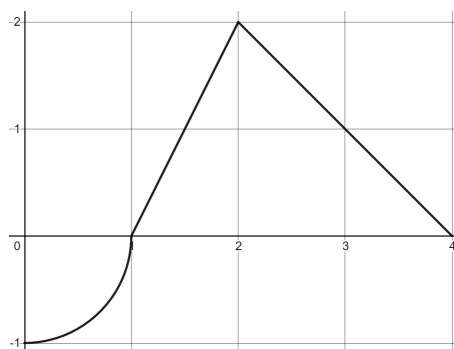
Name:_____

You are to complete these questions in 15 minutes. No notes, calculators, or any other electronic devices are allowed.

Please show all working.

1. Consider the following function $f(x)$.

   Calculate the following integrals:

   (a) $\displaystyle\int_0^1 f(x)\,dx$

   (b) $\displaystyle\int_1^4 f(x)\,dx$

   (c) $\displaystyle\int_4^0 f(x)\,dx$

   Solution:

   (a) $-\frac{\pi}{4}$

   <div align="right">(Answer +1)</div>

   (b) 3

   <div align="right">(Answer +1)</div>

   (c) $\frac{\pi}{4} - 3$

   <div align="right">(Answer +2; if $3 - \frac{\pi}{4}$, then +1)</div>
   <div align="right">error carried forward</div>

2. Calculate the definite integral

$$\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} (\sin x + \cos x)\, dx$$

Solution 1:

$$\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} (\sin x + \cos x)\, dx = \left[-\cos x + \sin x\right]_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \qquad \text{(Correct integral } +2\text{)}$$

$$= (-0 - 1) - (-0 + 1) \qquad \text{(Correct substitution } +1\text{)}$$
$$= -2 \qquad\qquad\qquad\qquad\quad \text{(Answer } +1\text{)}$$

Solution 2:

$$\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} (\sin x + \cos x)\, dx = \int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \sin x\, dx + \int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \cos x\, dx \qquad \text{(Split } +1\text{)}$$

$$= 0 + \int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \cos x\, dx \qquad\qquad \text{(Getting 0, } +2\text{)}$$

$$= -2 \qquad\qquad\qquad\qquad\qquad\quad \text{(Answer } +1\text{)}$$

3. State parts I and II of the Fundamental Theorem of Calculus. (You can just write the equation for each part.)

Solution 2:

For constant c,

$$\frac{d}{dx}\left(\int_c^x f(t)\, dt\right) = f(x).$$

$$\text{(Answer } +1\text{)}$$

Let $F' = f$. For constants a, b,

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

$$\text{(Answer } +1\text{)}$$

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