

Math 221 Sec 003 Quiz 9

Please show all working.

1. Find the derivative of the following function.

$$f(x) = \ln(e^{x^2} + 1)$$

Solution:

$$\begin{aligned} f'(x) &= \frac{1}{e^{x^2} + 1} \cdot 2xe^{x^2} && \text{(derivative of } \ln +1) \\ & && (+1 \text{ for each chain rule } +2) \\ &= \frac{2xe^{x^2}}{e^{x^2} + 1} && \text{(Correct answer } +1) \end{aligned}$$

2. Compute the following integral.

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} \cdot dx$$

Solution: Let $u = \sin x$.

(letting $u = \sin x +1$)

Then, we get $du = \cos x \, dx$.

$$dx = \frac{du}{\cos x}$$

(correct du , or $dx +1$)

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} \cdot dx &= \int_0^1 \frac{\cos x}{1 + u^2} \frac{du}{\cos x} \\ &= \int_0^1 \frac{1}{1 + u^2} du \\ &= \arctan(u) \Big|_0^1 && \text{(arctan } +1) \end{aligned}$$

(change of bounds +1 OR

subbing back $\sin x +1$)

(Do not award this point if their bounds
are the same after changing to du)

$$= \arctan(1) - \arctan(0) \quad \text{(correct substitution } +1)$$

$$= \frac{\pi}{4} \quad \text{(correct answer } +1)$$