Math 221 - Week 1 - Worksheet 2
Topics: Section 1.5 - The Limit of a Function

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Make an educated guess of the value of the following limits.

   (a) $\lim_{s\to 5} s - 3$.

   (b) $\lim_{u\to -2} u^2 - \cos(\pi u)$.

   (c) $\lim_{v\to 4} \dfrac{v+3}{4v-2}$.

2. Sketch the graph of an example of a function f that satisfies all of the following: $\lim_{x\to -3^-} f(x) = 2$, $\lim_{x\to -3^+} f(x) = 2$, $\lim_{x\to 1^-} f(x) = 4$, $\lim_{x\to 1^+} f(x) = -1$, $f(-3) = 4$, $f(1) = -1$.

3. Determine the infinite limit.

   (a) $\lim_{s\to 1^-} \dfrac{s^2-4}{s-1}$.

   (b) $\lim_{u\to 3^+} \dfrac{u^2-2u-8}{u^2-6u+9}$.

   (c) $\lim_{t\to 9^-} \dfrac{\sqrt{t}}{(t-9)^3}$.

   (d) $\lim_{\theta\to \pi^+} \dfrac{\theta-4}{\sin(\theta)}$

4. Consider the function $f(x) = \dfrac{2x-3}{(x-2)(x+4)}$.

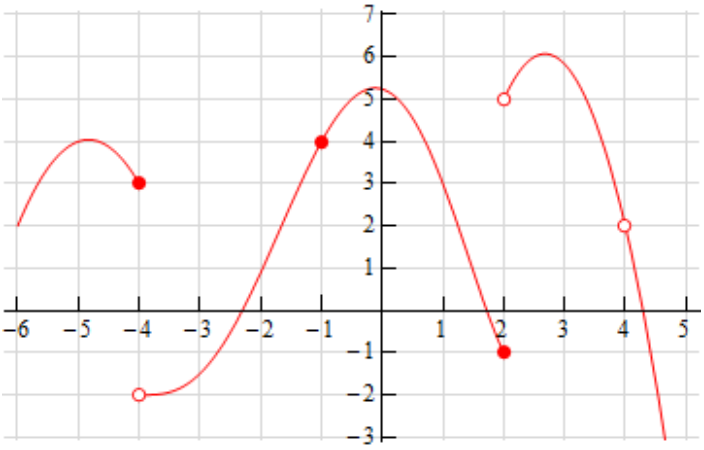
   (a) Find all the vertical asymptotes of f.

   (b) Compute $\lim\limits_{x\to 2^+} f(x)$, $\lim\limits_{x\to 2^-} f(x)$, $\lim\limits_{x\to -4^+} f(x)$, and $\lim\limits_{x\to -4^-} f(x)$.

   (c) Make a rough sketch of the function.

5. Consider the functions $f(x) = x + 2$, $g(x) = \dfrac{(x-3)(x+2)}{x-3}$, and $h(x) = \begin{cases} \dfrac{(x-3)(x+2)}{x-3} & x \neq 3 \\ 8 & x = 3. \end{cases}$. Sketch each of the functions. Then determine the limit as $x \to 3$ of each of the functions. If the limit does not exist, state so.

6. Below is the graph of $g(t)$. For each of the given points determine the value of $g(a)$, $\lim_{t\to a^-} g(t)$, $\lim_{t\to a^+} g(t)$, and $\lim_{t\to a} g(t)$. If any of the quantities do not exist, explain why.



(a) $a = -4$

(b) $a = -1$.

(c) $a = 2$.

(d) $a = 4$.

7. Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 3 + x & x < -2 \\ x^2 - 2 & -2 \le x \le 3 \\ 10 - x & x > 3. \end{cases}$$

8. Consider the function $f(x) = \tan\left(\frac{1}{x}\right)$.

(a) Show that $f(x) = 0$ for $x = \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \ldots$.

(b) Show that $f(x) = 1$ for $x = \frac{4}{\pi}, \frac{4}{5\pi}, \frac{4}{9\pi}, \ldots$.

(c) What can you conclude about $\lim_{x \to 0^+} \tan\left(\frac{1}{x}\right)$?