Math 221 - Week 10 - Worksheet 1
Topics: Section 4.3 - The Fundamental Theorem of Calculus

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Calculate the following using the FTC Part I when appropriate.

   (a) $\dfrac{d}{dx}\left(\displaystyle\int_0^x \sqrt{1-t^2}dt\right)$

   (b) $\dfrac{d}{dx}\left(\displaystyle\int_{-5}^x t^3 - 2t^2 + 1 \; dt\right)$

   (c) $\dfrac{d}{dx}\left(\displaystyle\int_2^7 t^2 dt\right)$

2. In this problem we will use the FTC to evaluate $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^3} t^2 \; dt\right)$

   (a) Define $F(u) = \displaystyle\int_0^u t^2 \; dt$. Use FTC to find $F'(u)$.

   (b) Let $u(x) = x^3$. Explain why $\frac{d}{dx}F(u) = F'(x^3) \cdot 3x^2$.

   (c) What is $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^3} t^2 \; dt\right)$?

3. Compute $\frac{d}{dx}\left(\int_2^{1/x} \arctan t \; dt\right)$

4. Let $F(x) = \int_2^x \frac{\cos(\sin(t^2))}{t^3} dt$. Compute $F'(x)$.

5. Compute $\left(\int_{\frac{1}{x^2}}^0 \cos(t^4 \sin(t)) dt\right)'$.

6. Compute $\frac{d}{dx}\int_{\pi x}^{\cos(x)} \frac{1}{1+t^3} dt$.

7. On what interval is the function $F(x) = \int_2^x \frac{1}{1+t+t^2} dt$ concave up? Concave down? Find the x-coordinates of any inflection points.

8. Let $f(x) = \frac{1}{2}x - 1$.

   (a) Sketch the graph of $f(x)$ on the interval $[0, 3]$. Use your picture to calculate the area under the curve on this interval.

   (b) Find a function $F(x)$ so that $F'(x) = f(x)$ ($F(x)$ is an antiderivative of $f(x)$).

   (c) Calculate $F(3) - F(0)$.

9. Evaluate the following definite integrals.

   (a) $\int_1^4 2x^4 - 3x^2 dx$

   (b) $\int_0^4 x\sqrt{x^3} dx$

   (c) $\int_0^{\frac{\pi}{4}} \sin(x) dx$

   (d) $\int_0^1 (x^3 - 1)^2 dx$

(e) $\int_{-1}^{3} |x-2| dx$

(f) $\int_{-1}^{5} (1+3x) dx$

(g) $\int_{0}^{2} (2-x^2) dx$

10. Compute $\int_{-1}^{1} x + x^3 dx$. Does your answer make sense geometrically?

11. Find $\int_0^5 f(x)\,dx$ if

$$f(x) = \begin{cases} 3, & x < 3 \\ x, & x \geq 3 \end{cases}$$

12. Find the area under the curve $y = \sqrt{4x+4}$ and above the x-axis between the vertical lines $x = 0$ and $x = 2$. Sketch a graph of the curve and the area.

13. Find two functions $F_1(x), F_2(x)$ such that $F_1'(x) = F_2'(x) = 4x - \cos(x)$. Use both of them to compute $\int_0^\pi 4x - \cos(x)dx$. Do you get the same answer?

14. The velocity (in meters per second) of a particle moving along a line is given by $v(t) = t^2 - 2t - 3$ for $2 \leq t \leq 4$. Find the displacement of the particle and the distance traveled by the particle during the given time interval.