Math 221 - Week 12 - Worksheet 2

Topics: Section 6.6 - Inverse Trigonometric Functions

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Determine the derivatives of the following functions:

   (a) $f(x) = \sin^{-1}(4x^2)$

   (b) $g(s) = \cos^{-1}(s)\ln(2s)$.

   (c) $y = (\tan^{-1} x)^2$

   (d) $f(x) = \arcsin(e^x)$

   (e) $y = \arctan\sqrt{\frac{1-x}{1+x}}$

   (f) $y = \tan^{-1}\left(\frac{x}{a}\right) + \ln\sqrt{\frac{x-a}{x+a}}$

2. Find the absolute max and absoulte min of the function $f(x) = e^x - ex$ on the interval $0 \leq x \leq 5$.

3. Find y' if $\tan^{-1}(x^2 y) = 2x + xy$.

4. Find and equation of the tangent line to the curve $y = 3\arccos(x/2)$ at the point $(1, \pi)$.

5. Show that there is exactly one root of the equation $\ln(x) = 3 - x$ and that it lies between 1 and e.

6. Evaluate the following integrals.

   (a) $\int \frac{1}{(y-1)^2+1} dy$

(b) $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$

(c) $\int \frac{1+x}{1+x^2} dx$

(d) $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

(e) $\int \frac{dx}{\sqrt{1-x^2}\sin^{-1} x}$

(f) $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

(g) $\displaystyle\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx$

(h) $\displaystyle\int \frac{x}{1+x^4}dx$

(i) $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}dx$ for $a > 0$

(j) $\displaystyle\int \frac{\sin(\arctan(x))}{2+2x^2}dx$

7. Find $\frac{dq}{dp}$ if $\arcsin(pq) + q^2 = \frac{q}{p}$.

8. Eliminate the trig functions from the following expressions:

   (a) $\tan(\sin^{-1} x)$

   (b) $\sin(\tan^{-1} x)$

   (c) $\sin(2 \arccos x)$

9. If $g(x) = x \sin^{-1}(x/4) + \sqrt{16 - x^2}$, find the equation of the line tangent to $g(x)$ at $x = 2$.

10. Sketch the function $f(x) = \tan^{-1}(x) - x$ using the techniques you learned in Chapter 3.