Math 221 - Week 13 - Worksheet 2

Topics: Section 5.1 - Areas Between Curves and Section 5.2 - Volumes

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. We will find the area bounded by the curves $x = 3 - y^2$ and $x = -1$.

   (a) Sketch the two curves, and find the points where they intersect.

   (b) Write down two different integrals which represent the area (one should be in terms of x, and one should be in terms of y).

   (c) Evaluate both integrals to check that they give the same answer.

2. Consider the curves $y = x^2$ and $y = \sqrt{x}$

   (a) Find the points (x, y) where the curves intersect. Sketch the curves.

(b) Write down an integral which represents the area between the two curves.

(c) Solve each equation for x in terms of y. If you have to choose $\pm$, choose the one that makes sense for the area we are interested in.

(d) Write down an integral in terms of y which represents the area between the two curves.

3. Let $f(x) = x^2$, and let (a, a^2) be a point on the curve (assume $a > 0$).

   (a) Find the equation of the tangent line to $f(x)$ at (a, a^2).

   (b) Sketch the curve, the point, and the tangent line. Shade the area bounded by the x-axis, the curve, and the line.

   (c) If the shaded area is $\frac{2}{3}$, what is the value of a?

4. Let $f(x) = \sin x$, $g(x) = \cos x$.

(a) Find the points (x, y) where the functions intersect on the interval $[0, \frac{\pi}{2}]$

(b) Sketch these two functions on the interval $[0, \frac{\pi}{2}]$. Label the points where they intersect.

(c) Write down an integral (or sum of integrals) which represent the area enclosed by $f(x)$, $g(x)$, $x = \frac{\pi}{2}$, and the y-axis.

5. Determine the area bounded by the given equations.

(a) $y = \frac{8}{x}$, $y = 2x$, $x = 4$

(b) $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$, $y = -2$

(c) $y = e^{1+2x}$, $y = e^{1-x}$, $x = -2$, $x = 1$

(d) $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, $x = 5$

(e) $x = -y^2 + 10$, $x = (y - 2)^2$

(f) $y = xe^{-x^2}$, $y = x + 1$, $x = 2$, y-axis.

6. Consider the region between $y = x^2$, $x = 2$, and the x-axis. Let's call this region A.

(a) Find the volume of the shape whose base is A and whose vertical cross sections are semicircles.

(b) Find the volume of the shape whose base is A and whose horizontal cross sections are semicircles.

7. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are equilateral triangles (Hint: The area of an equilateral triangle with side length s is $\frac{\sqrt{3}s^2}{4}$).

8. Find, using integration, a formula for the volume of a triangular pyramid, with base an equilateral triangle with side lengths a, and height h.