

Math 221 - Week 14 - Worksheet 1
Topics: Section 5.2 - Volumes and Final Review

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Find the volume of the solid obtained by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the x -axis.

2. Find the volume of the solid obtained by revolving the region enclosed by $x = \sqrt{2 \sin(2y)}$, $0 \leq y \leq \pi/2$, and $x = 0$ about the y -axis.

3. Find the volume of the solid obtained by revolving the region bounded by $y = \sqrt{\cos(x)}$, $0 \leq x \leq \pi/2$, $y = 0$ and $x = 0$ about the x -axis.

4. Write down an integral that represents the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 2 - x$ about the x -axis.

5. Write down an integral that represents the volume of the solid obtained by revolving the region bounded by $y = x^2$ and the line $y = 1$ about the line $y = -2$.

6. Write down an integral that represents the volume of the solid obtained by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the line $x = 4$.

Final Exam Review

7. A rectangle is to be inscribed under the arch of the curve $y = 1 - x^2$ from $x = -1$ to $x = 1$. What are the dimensions of the rectangle with largest area, and what is the largest area?

8. Compute the first derivative of the following functions

(a) $f(x) = \ln(x^3 - 4x) \sin(2x)$

(b) $g(s) = \sin(\cos(e^{\sin(s)}))$

(c) $h(t) = \sqrt{\frac{t-1}{t^2+2}}$

(d) $g(t) = \frac{\cos(2t)}{t-5}$

(e) $F(x) = \int_3^{x^3} e^{4t^2} dt.$

9. You are videotaping a race from the inside of a blimp 132 ft directly above the finish line, following a car that is moving at 180 mph (264 ft/s). How fast will your camera angle be changing when the car is finishing the race?

10. Let $f(x) = 3x^2 + 4x$. Use the definition of the derivative to compute $f'(2)$.

11. Compute the following limits.

(a) $\lim_{\theta \rightarrow 0} \frac{\tan(\theta)}{\theta + \sin(\theta)}$

(b) $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$.

(c) $\lim_{u \rightarrow 0} \frac{5 - 5 \cos(u)}{e^u - u - 1}$.

(d) $\lim_{x \rightarrow \infty} \left(\frac{e^x + 1}{e^x - 1} \right)^{\ln(x)}$

12. Compute the following integrals

(a) $\int e^x \sin(e^x) dx$.

(b) $\int_{-1}^1 \frac{dx}{3x - 4}$.

(c) $\int_1^3 \frac{s^2 + 2\sqrt{s} - s + 3}{4s}$.

13. Sketch the graph of the function $f(x) = \frac{x^2-4}{2x}$.