Math 221 - Week 2 - Worksheet 1
Topics: Section 1.6 - Calculating Limits Using the Limit Laws

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Given that $\lim_{x\to -4} f(x) = 6$, $\lim_{x\to -4} g(x) = 0$, and $\lim_{x\to -4} h(x) = 1$, find the limits below. If the limit does not exist, explain why.

   (a) $\lim_{x\to -4} [f(x) - 4h(x)]$

   (b) $\lim_{x\to -4} [f(x)]^3$

   (c) $\lim_{x\to -4} \dfrac{g(x)}{3h(x)}$.

   (d) $\lim_{x\to -4} \dfrac{h(x)}{2g(x)}$.

2. Evaluate each limit and justify each step by indicating the appropriate Limit Laws.

   (a) $\lim_{a\to 2} \dfrac{a^4 - 8a + 4}{3a^2 + 16}$

   (b) $\lim_{u\to -1} \sqrt{\dfrac{2u+5}{3u+11}}$.

3. Evaluate the following limit, if it exists. If the limit does not exist, explain why. If you use a theorem, clearly state which theorem you are using.

   (a) $\displaystyle\lim_{x\to -6} \frac{\frac{1}{x}+\frac{1}{6}}{x+6}$

   (b) $\displaystyle\lim_{x\to 1} \frac{x^3-1}{x-1}.$

   (c) $\displaystyle\lim_{v\to \frac{1}{2}^-} \frac{|2v-1|}{2v-1}.$

   (d) $\displaystyle\lim_{v\to \frac{1}{2}} \frac{|2v-1|}{2v-1}.$

(e) $\lim_{x \to 0} x^4 \cos\left(\frac{1}{x}\right)$

(f) $\lim_{u \to -3} \frac{2 - \sqrt{u^2 - 5}}{u + 3}$ (Hint: multiply by the conjugate).

(g) $\lim_{t \to 0} t^2 2^{\sin\left(\frac{1}{t^2}\right)}$.

(h) $\lim_{t \to 7} \frac{\sqrt{t+2} - 3}{t - 7}$

(i) $\lim_{h\to 0} \frac{(4+h)^2 - 16}{h}$.

(j) $\lim_{x\to 4} (x-4)^2 \sin\left(\frac{\cos(x)}{x-4}\right)$.

4. Is there a number a such that $\lim_{x\to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists? If so, find the value of a and the value of the limit.

5. True or False.

(a) If $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5} \frac{f(x)}{g(x)}$ does not exist. If the answer is false, give a counterexample (that is, an example that satisfies the hypothesis but not the conclusion).

(b) if $f(x) > 1$ for all x and if $\lim_{x \to 0} f(x)$ exists, then $\lim_{x \to 0} f(x) > 1$. If the answer is false, give a counterexample.

(c) If $\lim_{x \to 6} f(x)g(x)$ exists, then the limit must be $f(6)g(6)$. If the answer is false, give a counterexample.

(d) If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$, then $\lim_{x \to 0} [f(x) - g(x)] = 0$. If the answer is false, give a counterexample.

(e) $\lim_{x \to 4} \left(\frac{2x}{x-4} - \frac{8}{x-4} \right) = \lim_{x \to 4} \frac{2x}{x-4} - \lim_{x \to 4} \frac{8}{x-4}$.