Math 221 - Week 2 - Worksheet 2

Topics: Section 1.7 - The Precise Definition of a Limit

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

The goal of today's worksheet is to help you understand and learn the following definition:

The Precise Definition of a Limit We say that $\lim_{x \to a} f(x) = L$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

1. Write the given limit using the precise definition of limits.

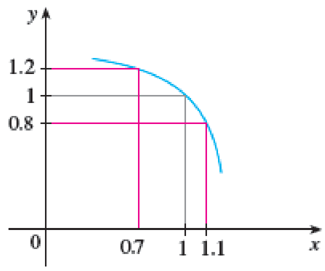
   (a) $\lim_{x \to 1} x + 3 = 4$.

   (b) $\lim_{x \to -3} x^2 - 4 = 5$.

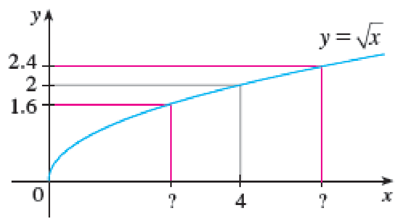
   (c) $\lim_{u \to -2\pi} \sin(u) = 0$.

   (d) $\lim_{v \to b} g(v) = M$.

2. Use the given graph of f to find a number δ such that if $|x-1|<\delta$ then $|f(x)-1|<0.2$.



3. Use the given graph of $f(x)=\sqrt{x}$ to find a number δ such that if $|x-4|<\delta$ then $|\sqrt{x}-2|<0.4$.



4. Look carefully at the following statements. Some of the statements are correct, and some have one or more issues. For each of the statements, decide whether it is correct or not. If it is not correct, identify the issue(s) and correct it (them).

   (a) We say that $\lim_{x\to b} f(x)=L$ if for every number $\epsilon>0$ there is a number $\delta>0$ such that if $0<|x-b|<\epsilon$ then $|f(x)-L|<\delta$.

   (b) We say that $\lim_{x\to a} f(x)=L$ if for every number $\epsilon>0$ there is a number $\delta>0$ such that if $0\le|x-a|<\delta$ then $|f(x)-L|<\epsilon$.

(c) We say that $\lim_{x \to a} f(x) = L$ if for every number $M > 0$ there is a number $N > 0$ such that if $0 < |x - a| < N$ then $|f(x) - L| < M$.

(d) We say that $\lim_{u \to c} f(u) = L$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if $0 < |f(u) - c| < \delta$ then $|u - L| < \epsilon$.

5. Let $f(x) = 3x - 6$. Note that $\lim_{x \to 1} f(x) = -3$. Recall that the distance between two numbers a, b is given by $|b - a.|$.

(a) How close does x need to be to 1 so that $f(x)$ is as most 2 away from -3 (when $x \neq 1$)?

(b) How close does x need to be to 1 so that $f(x)$ is as most 0.3 away from -3 (when $x \neq 1$)?

(c) How close does x need to be to 1 so that $f(x)$ is as most 0.02 away from -3 (when $x \neq 1$)?

(d) How close does x need to be to 1 so that $f(x)$ is as most ϵ away from -3 (when $x \neq 1$)?

(e) Find δ, in terms of ϵ, so that when $0 < |x - 1| < \delta$, we have $|f(x) - (-3)| < \epsilon$.

6. Find a number δ such that if $|x-3|<\delta$, then $|3x-9|<0.01$.

7. Given that $\lim_{x\to-2}(2x-7)=-11$, find δ so that when $0<|x+2|<\delta$, we have $|2x-7+11|<0.1$.

8. Consider the function $f(x)=x^3$.

   (a) Determine $\lim\limits_{x\to 0} f(x)$ and sketch the graph of $f(x)$.

   (b) For $\epsilon=2$, determine δ such that $|x-0|<\delta$ implies $|f(x)-L|<\epsilon$ by considering the problem graphically. Draw the "windows" on your graph.

   (c) Repeat part (b) with $\epsilon=1$. Make a prediction for how we can choose δ if we are given an arbitrary ϵ. (Hint: your δ should depend on ϵ).

9. Let $f(x) = 5x + 3$. It is known that $\lim_{x \to 1} f(x) = 8$. Given $\epsilon > 0$ find $\delta > 0$, depending on ϵ, such that $|f(x) - 8| < \epsilon$ when $0 < |x - 1| < \delta$.

10. Let $f(x) = -4x + 2$. It is known that $\lim_{x \to 2} f(x) = -6$. Given $\epsilon > 0$ find $\delta > 0$, depending on ϵ, such that $|f(x) + 6| < \epsilon$ when $0 < |x - 2| < \delta$.

11. Let $f(x) = -7x - 4$. It is known that $\lim_{x \to -1} f(x) = 3$. Given $\epsilon > 0$ find $\delta > 0$, depending on ϵ, such that $|f(x) - 3| < \epsilon$ when $0 < |x + 1| < \delta$.

12. Let $f(x) = 3x + 1$. It is known that $\lim_{x \to -2} f(x) = -5$. Given $\epsilon > 0$ find $\delta > 0$, depending on ϵ, such that $|f(x) + 5| < \epsilon$ when $0 < |x + 2| < \delta$.