Math 221 - Week 3 - Worksheet 1

Topics: Section 1.8 - Continuity

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. State the definition of continuity.

2. True or False: If $\lim_{x \to 0} f(x)$ exists, then $f(x)$ is continuous at $x = 0$. (If the statement is true, explain why. If the statement is false, come up with a counterexample.)

3. Draw a graph of a function $h(t)$ that satisfies all of the following properties.
   (a) The domain of h is all real numbers and the range of h is all positive real numbers.
   (b) $h(t)$ is not continuous at $t = 1$ and at $t = 4$.
   (c) $\lim_{t \to 1^+} h(t). = 2$ and $\lim_{t \to 1^-} h(t) = 2$.
   (d) $\lim_{t \to 4^+} h(t). = 1$ and $\lim_{t \to 4^-} h(t) = 3$.

4. Consider the function $g(x) = \begin{cases} x & x < -2 \\ bx^2 & x \geq -2 \end{cases}$ where b is some constant.

(a) Compute $\lim_{x \to -2^-} g(x)$.

(b) Compute $\lim_{x \to -2^+} g(x)$.

(c) Compute $g(-2)$.

(d) For what value of b will $\lim_{x \to -2} g(x)$ exist?

5. Let

$$g(x) = \begin{cases} ax + 2 & x < -1 \\ x^2 + b & -1 \leq x \leq 2 \\ 2x + 4 & x > 2. \end{cases}$$

Find the values of a and b that make g continuous everywhere.

6. Locate the discontinuities of the function $y(x) = \dfrac{4}{1 + \cos(x)}$.

7. Use the Intermediate Value Theorem to show that there exists c in $[0, 1]$ such that $f(c) = 0$, where $f(x) = -8x^4 + 2x^3 - x + 1$.

8. Consider the function $f(x) = \dfrac{x^2 - 1}{x - 1}$. How would you "remove the discontinuity" of f? In other words, how would you define $f(1)$ in order to make f continuous at 1?

9. Consider the function $f(x) = \dfrac{x^2 + 6x + 8}{x + 2}$. How would you "remove the discontinuity" of f?

10. Suppose $y = h(x)$ is the equation of a line. Find the equation for $h(x)$ if we are given that the following function $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} \dfrac{2x^2+6x+4}{3x^2-3} & x < -1 \\ h(x) & -1 \leq x \leq 3 \\ \dfrac{6}{x^2-9} - \dfrac{1}{x-3} & x > 3 \end{cases}$$

11. Show that there exists an intersection point between the graphs of $y = \sin(x)$ and $y = 4^{x/\pi}$ in the interval $\left(\dfrac{-3\pi}{2}, 0\right)$.

12. Suppose f is continuous on $[2, 8]$ and the only solution of the equation $f(x) = 4$ are $x = 3$ and $x = 7$. If $f(4) = 6$, explain why $f(5) > 4$.

13. Find the equation of the line through the points $(2, 4)$ and $(1, -2)$.

14. Let $f(x) = \sqrt{x}$.

(a) Find the slope of the line through the points $(4, f(4))$ and $(9, f(9))$.

(b) Find the slope of the line through the points $(a, f(a))$ and $(b, f(b))$.