Math 221 - Week 3 - Worksheet 2
Topics: Section 2.1 - Derivatives and Rate of Change

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Let f be a function. Find the equation of the slope of the secant line that passes through the points $(a, f(a))$ and $(a+h, f(a+h))$.

2. Let f be a function. Find the equation of the slope of the secant line that passes through the points $(a, f(a))$ and $(b, f(b))$.

3. Let $f(x) = 2x^2$.

   (a) Find the slope of the line through the points $(1, f(1))$ and $(2, f(2))$.

   (b) Find the slope of the line through the points $(a, f(a))$ and $(b, f(b))$.

   (c) Compute $\lim_{b \to 1} \dfrac{f(b) - f(1)}{b - 1}$.

(d) Write the equation of the line tangent to $f(x) = 2x^2$ at $x = 1$.

(e) Sketch the function $f(x) = 2x^2$ and the tangent line to the function at $x = 1$.

4. Use the definition of the derivative to find the derivative of the function $f(x) = 3x^2 + 4$ at the point $x = 2$.

5. Use the definition of the derivative to find the derivative of the function $f(x) = \dfrac{1}{x-2}$ at the point $x = -1$.

6. Use the definition of the derivative to find the derivative $f'(6)$ where $f(x) = \sqrt{x-4}$.

7. Using the definition of the derivative, compute $g'(1)$ where $g(x) = x^2 - 3x$.

8. Consider the function $y = \dfrac{3}{2+x}$.

   (a) Using the definition of the derivative, compute the slope of the tangent line to the function $y = \dfrac{3}{2+x}$ at the point $(-1, 3)$.

   (b) Find the equation of the line tangent to $y(x)$ at $x = -1$.

9. Let $f(x) = x^3 - 3x + 1$. Using the definition of the derivative, compute the slope of the tangent line at the point $(a, f(a))$. Where is the tangent line horizontal? Use this to sketch a graph of $y = f(x)$.

10. Suppose the position of a car is given by the function $s(t) = t - t^2$ for $t \geq 0$.

    (a) Find the average velocity of the car from $t = 0$ to $t = \frac{1}{2}$.

    (b) Find the instantaneous velocity of the car at time $t = 1$.

    (c) At what time is the car stopped?