Math 221 - Week 4 - Worksheet 1
Topics: Section 2.2 - Derivatives and Rate of Change, Section 2.3 - Differentiation Formulas

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Let $f(x) = 2x^2 + x + 1$.

   (a) Compute the derivative $f'(x)$ using the definition of the derivative.

   (b) Your friend says that the equation for the tangent line to $f(x)$ at the point $(1, 4)$ is

   $$y - 4 = (4x + 1)(x - 1).$$

   What did they do wrong?

2. Use the definition of the derivative to find the derivative $f'(x)$ where $f(x) = \sqrt{x + 1}$.

3. Let $f(x) = x + |x|$. What is $f'(c)$ for $c > 0$?. What is $f'(c)$ for $c < 0$?. What about $f'(0)$?

4. Is the function

$$f(x) = \begin{cases} 0 & : x \leq 0 \\ x^2 & : x > 0 \end{cases}$$

continuous at $x = 0$? Is it differentiable at $x = 0$?

5. For which values of a and b is the following function differentiable at $x = 1$? Sketch a graph for those values of a and b.

$$f(x) = \begin{cases} ax^2 + b & : x < 1 \\ x - x^2 & : x \geq 1 \end{cases}$$

6. Let $f(x) = x + 2$, $g(x) = 2x - 1$.

   (a) Compute $f'(x)$ and $g'(x)$.

   (b) Compute $[f(x)g(x)]'$. How does it compare to $f'(x)g'(x)$?

7. Let f, g be functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = -5$, and $g'(2) = 2$. Use differentiation rules to find $h'(2)$ for

   (a) $h(x) = 3f(x) - g(x)$

   (b) $h(x) = f(x)g(x)$

(c) $h(x) = \frac{1}{f(x)}$

(d) $h(x) = \frac{g(x)}{f(x)}$

8. Compute the derivatives of the following functions:

(a) $f(x) = 4\pi^2$

(b) $f(x) = x^3 + 2x + 4$

(c) $f(x) = \frac{x^2-2x+1}{\sqrt{x}}$

(d) $f(x) = \frac{2x-1}{3x+2}$

(e) Compute $g'(r)$ and $g''(r)$ for $g(r) = \left(\frac{1}{r^2} - \frac{3}{r^4}\right)(r + 5r^3)$.

(f) Find the first and second derivatives of $f(t) = (1 - 7t)^2$.

9. Suppose $f(x)$ is a function which passes through the point $(4, 3)$, and that the line tangent to $y = f(x)$ at $(4, 3)$ also passes through the point $(0, 2)$.

   (a) Sketch the tangent line along with a *possible* graph of $f(x)$ (make sure to label the two given points).

   (b) Find an equation of the tangent line you drew.

   (c) What is $f(4)$? What is $f'(4)$?

10. Let $f(x) = \dfrac{x-1}{x+1}$. What is $(x+1) \cdot f(x)$? Can you use this to come up with a formula for $f'(x)$ without using the quotient rule?