Math 221 - Week 5 - Worksheet 1
Topics: Section 2.5 - The Chain Rule, Section 2.6 - Implicit Differentiation

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Each of the following functions can be written as a composition. Find $f(x)$ and $g(x)$ so that the following function is of the form $f(g(x))$. Then, find the derivative of the function using the chain rule.

   (a) $y = (2x+1)^2$. Can you take the derivative another way to check your work? How many ways can you take this derivative?

   (b) $y = \sin(4x)$

   (c) $y = \sqrt{2+x^2} + (2+x^2)^3$

   (d) $y(z) = \sqrt{\dfrac{z-1}{z+1}}$

2. A differentiable function f satisfies $f(3) = 5, f(\frac{1}{9}) = 7, f'(3) = 11$, and $f'(\frac{1}{9}) = 13$. Let $g(x) = f(\frac{1}{x^2})$. Find the equation to the tangent line of $y = g(x)$ when $x = 3$.

3. Suppose $f(x)$ and $g(x)$ are functions whose values and derivatives at $x = 0$ and $x = 1$ are given in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$1/3$
1	3	-4	$-1/3$	$-8/3$

Define:

$v(x) = f(g(x))$ $\qquad w(x) = g(f(x))$ $\qquad p(x) = f(x)g(x)$ $\qquad q(x) = \frac{g(x)}{f(x)}$.

Determine $v(0), w(0), p(0), q(0)$ and $v'(0), w'(0), p'(0), q'(0)$. If there isn't enough information to compute a value, state so.

4. Compute the derivative of $y = \tan(\sqrt{1 + x^2})$.

5. Compute the derivative of $h(x) = \left(\frac{\sec(x)}{1+x^2} \right)^{1/3}$.

6. Compute the derivative of $y = \tan((\sin(2x))^2)$.

7. Find $\frac{d^2y}{dx^2}$ if $y = (3x + \tan(x))^3$.

8. Consider the curve defined by $x^2 + y^2 = 1$.

   (a) Is y a function of x? How can you see this graphically? How can you see this algebraically?

(b) Solve for y as a function of x on the bottom half of the circle. Find y'.

(c) Solve for y as a function of x on the top half of the circle. Find y'.

(d) Use implicit differentiation to find y'. How does this answer relate to your answer from the previous parts?

9. Use implicit differentiation to find $\frac{dy}{dx}$ if $y^2 + xy = 4x^2$.

10. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $\cos\left(\dfrac{x}{y}\right) = x\sin(y)$.

11. Find the equation of the tangent line to the curve defined by $\cos(x^2y) = 3xy^2 + y$ at the point $(0, 1)$.

12. Find a point on the curve $x^2 = y^3 + y + 2$ and use implicit differentiation to find the equation of the tangent line through that point.