Math 221 - Week 6 - Worksheet 1

Topics: Midterm 1 Review Worksheet - Sections 1.5, 1.6, 1.7, 1.8, 2.1, 2.2, 2.3, 2.4, 2.5

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Evaluate the limit, if it exists. If the limit does not exist, state whether it is ∞, $-\infty$, or neither. You may NOT use L'Hopitals rule. Show your work. If you use a theorem, clearly state which theorem you are using.

   (a) $\lim_{\theta \to 0} \dfrac{\sin(4\theta)\sin(5\theta)}{\theta^2}$

   (b) $\lim_{u \to 0} u^6 \sin\left(\dfrac{4^u}{6u^4}\right)$

   (c) $\lim_{x \to 0} \left(\dfrac{1}{x^4} - \dfrac{1}{x^4\sqrt{1+x^4}}\right)$

   (d) $\lim_{x \to 5} g(x)$, where $g(x) = \begin{cases} \dfrac{3x-15}{x-5} & \text{if } x \neq 5 \\ 4 & \text{if } x = 5 \end{cases}$

   (e) $\lim_{t \to 6^-} \dfrac{t^2+4t-7}{t-6}$

   (f) $\lim_{t \to -3} \dfrac{4t+12}{|t+3|}$

   (g) $\lim_{x \to 0} x^2(4^{\cos\left(\frac{1}{x^3}\right)} + 3\pi x - 2)$.

   (h) $\lim_{\theta \to 0} \dfrac{\tan(4\theta)}{8\theta}$

2. Is there a number a such that $\lim_{x\to 5} \dfrac{x^2 - x - ax - 2a + 1}{x^2 - 3x - 10}$ exists? If so, find the value of a and the value of the limit.

3. Consider the function $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 1}$.

   (a) Identify all values of x for which the function is discontinuous.

   (b) Does the function have a removable discontinuity? If so, for which value(s) of x?

   (c) Does the function have a jump discontinuity? If so, for which value(s) of x?

   (d) For any removable discontinuities, write down the corresponding continuous extension.

4. Does the function $f(x) = \sin(x) - 2x + 4$ have a root on the interval $[0, 2\pi]$? Explain why. If you use a theorem, clearly state which theorem you are using.

5. Find a number δ such that if $0 < |x - 5| < \delta$, then $|4x - 20| < \epsilon$, where $\epsilon = 0.1$.

6. Let $f(x) = 2x + 11$. We know that $\lim\limits_{x\to -1} f(x) = 9$. Given $\epsilon = 0.3$, find $\delta > 0$ such that $|f(x) - 9| < \epsilon$ when $0 < |x + 1| < \delta$.

7. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} ax^2 - bx & \text{if } x < -1 \\ 2x - a + bx & \text{if } -1 \le x \le 2 \\ \dfrac{x^2 - 4}{x - 2} & \text{if } x > 2. \end{cases}$$

8. Suppose $f(x)$ is continuous on $0 \le x \le 7$ and the only solutions of the equation $f(x) = 3$ are $x = 1$ and $x = 6$. If $f(4) = 2$, then which of the following options is correct?

   (a) $f(2) > 3$

   (b) $f(2) < 3$

   (c) it is not possible to determine whether $f(2) > 3$ or $f(2) < 3$ with the information provided.

   Explain why (if you use a theorem, clearly state which theorem you are using):

9. Let $f(x) = 3x^2 - x$.

   (a) Use the limit definition of the derivative to compute $f'(1)$.

   (b) What is the equation of the line tangent to the curve $y = f(x)$ at $x = 1$?

10. Let $f(x) = \dfrac{1}{2x - 3}$

    (a) Use the limit definition of the derivative to compute $f'(2)$.

    (b) What is the equation of the line tangent to the curve $y = f(x)$ at $x = 2$?

11. Compute the following derivatives. Use any method.

(a) Let $f(x) = 3x^5 \cos(x)$. Compute $f''(x)$ (that is, the second derivative of f).

(b) Let $y(x) = (x^3 + 4)^6$. Compute $y'(x)$.

(c) Let $g(t) = \frac{\sqrt{t}}{\cos(t)}$. Compute $g'(t)$.

(d) Let $h(s) = \sin(\cos(\sin(s^4 + 5)))$. Compute $h'(s)$.

(e) Let $r(t) = \sqrt{\sin(t)(2t^3 + 4t^2)^6 + 4}$. Compute $r'(t)$.

(f) Let $h(v) = 2\sin(v) - v^{-3} + \pi^4$. Compute $h'(v)$.

(g) Let $u(t) = \frac{\sin(4t^3)}{2\sqrt{5t}}$. Compute $u'(t)$.

(h) Let $f(t) = \sin(4t)$. Compute $f^{(2001)}(t)$, that is, the 2001 th derivative of f.