Math 221 - Week 7 - Worksheet 1

Topics: Section 3.3 - How Derivatives Affect the Shape of a Graph, Section 3.4 - Limits at Infinity; Horizontal Asymptotes

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Determine where $h(x) = x^3 + x^2 - x + 1$ is increasing or decreasing and its local extrema.

2. Sketch the graph of a function $f(x)$ such that
   (a) $f(3) = 5$ and $f(5) = 0$.
   (b) $f'(3) = f'(5) = 0$.
   (c) $f'(x) > 0$ if $x < 3$ or $x > 5$.
   (d) $f'(x) < 0$ if $3 < x < 5$.

3. Let $f(x) = x^4 - 4x^3$.
   (a) Find $f'(x)$ and $f''(x)$.

   (b) Determine where f is positive, negative, zero, or undefined.

(c) Determine where f' is positive, negative, zero, or undefined.

(d) Determine where f'' is positive, negative, zero, or undefined.

(e) Find any local extrema, and the intervals where f is increasing or decreasing.

(f) Find any inflection points of f, and the intervals where f is concave down (concave) or concave up (convex).

(g) Sketch the function.

4. Is it possible for a function $h(x)$ to satisfy $h'(x) = 0$ and $h''(x) > 0$ for every x? Why or why not?

5. Consider the function $g(t) = 4t - \cos^3(t)$.

   (a) Show that g has at least one zero (root). If you use a theorem, state explicitly which theorem you are using.

   (b) How many zeroes (roots) does g have? (Hint: what can you say about the sign of $g'(t)$?)

6. For the function $f(x) = \sin(x) + \cos(x)$, find all local extrema, intervals where f is increasing and decreasing, inflection points, and intervals where f is concave up and concave down on the region $[-2\pi, 2\pi]$.

7. Evaluate the following limits.

(a) $\lim_{x\to\infty} \dfrac{2x+1}{3x+4}$

(b) $\lim_{x\to\infty} \dfrac{x+3000}{2x^2-10}$

(c) $\lim_{x\to-\infty} \dfrac{x^2+1}{10^{100}x^2-x+1}$

(d) $\lim_{x\to-\infty} \dfrac{3x^2+4}{x-2}$

8. The limit laws we learned also apply to limits at infinity. That being said, what is wrong with the following?

$$1 = \lim_{x\to\infty} 1 = \lim_{x\to\infty} \frac{1}{x}\cdot x = \lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} x = 0 \cdot \lim_{x\to\infty} x = 0$$

9. Calculate $\lim_{x\to\infty} \frac{x^2+\cos(x)}{2x^2+4x+1}$.

10. Compute $\lim_{x\to-\infty} \sqrt{9x^2-x}+3x$.

11. Evaluate $\lim_{x\to\infty} \frac{4x+1}{\sqrt{x^2+2}}$

12. Evaluate $\lim_{x\to-\infty} \cos\left(\frac{\pi x^2+1}{4x^2-3}\right)$

13. Evaluate $\lim_{x\to\infty} \frac{x\sin(x)+x\cos(x)}{2x^2+3x-1}$

14. Let $f(x) = \dfrac{5x^2}{x^2 - 4}$. Find all asymptotes of $f(x)$ by evaluating all relevant limits.

15. Find all asymptotes of the function $f(x) = \dfrac{x^2 + x - 2}{x^2 - 1}$ by evaluating all relevant limits.

16. Find all asymptotes of the function $h(x) = \dfrac{x + 2}{\sqrt{x^2 + 1}}$.