Math 221 - Week 9 - Worksheet 1

Topics: Section 3.9 - Antiderivatives and Section 4.1 - Areas and Distances

Instructions: Listen to your TA's instructions. There are substantially more problems on this worksheet than we expect to be done in discussion, and your TA might not have you do problems in order. The worksheets are intentionally longer than will be covered in discussion in order to give students additional practice problems they may use to study. Do not worry if you do not finish the worksheet :).

1. Find the most general antiderivative of the following functions (don't forget the $+C$!).

   (a) $f(x) = 4x + \pi$

   (b) $h(\theta) = 4\sin(\theta) + \sec^2(\theta) + \theta^3$.

   (c) $g(s) = \sqrt{s} - 2s^{-3} + (s-3)^2$.

2. In each of the problems below, find f.

   (a) $f'(x) = \sqrt{x}(6 + x)$, with $f(1) = 10$.

   (b) $f''(x) = 20x^3 + 12x^2 + 4$, with $f(0) = 8$ and $f(1) = 5$.

(c) $f'''(x) = \cos(x)$, with $f(0) = 1$, $f'(0) = 2$ and $f''(0) = -3$.

3. Suppose you are traveling at 50mph.

(a) How far do you travel in 1/2 hour?

(b) If $v(t)$ represents your velocity at time t, then $v(t) = 50$ (the function is constant). Sketch $v(t)$ on the interval $[0, 1]$.

(c) What is the area under the curve of $v(t)$ on the interval $[0, \frac{1}{2}]$?

(d) How far do you travel after t hours? Write down a function $d(t)$ which represents the distance traveled after t hours. Sketch $d(t)$ on the interval $[0, 1]$. What is $d(\frac{1}{2}) - d(0)$?

(e) How do the functions $v(t)$ and $d(t)$ relate? (Hint: derivatives)

(f) In your sketch of $d(t)$, what is the slope of the tangent line at $t = \frac{1}{2}$? Does it equal $d'(\frac{1}{2})$? Does it equal $v(\frac{1}{2})$?

4. Evaluate the following sums.

(a) $\sum_{i=0}^{3} 2^i$

(b) $\sum_{n=1}^{4} f(n)$ where $f(x) = x^2$

5. (a) Estimate the area under the graph of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 5$ using four rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

(b) Repeat this for left endpoints.

6. (a) Write the area under the graph of $f(x) = x^3$ from $x = 0$ to $x = 3$ as the limit of a Riemann sum.

(b) Write $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \frac{1}{n}$ as an integral.

(c) Write the area under the graph of $f(x) = \frac{2x}{x^2+1}$ with $1 \leq x \leq 3$ as the limit of a Riemann sum.

7. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff? Assume the acceleration due to gravity is 32 ft/s^2.

8. (a) Find two functions which are not equal, but which have the same derivative.

(b) True or false: If $f(x)$ is a differentiable function on (a, b) with $f'(x) = 0$ for all x on (a, b), then $f(x)$ is constant.

(c) Suppose $f(x)$ and $g(x)$ are differentiable functions on an interval (a, b) which have the same derivative. What does this tell you about $\Big(f(x) - g(x)\Big)'$?

(d) Use the previous parts to show that if $f(x)$ and $g(x)$ are differentiable functions on an interval (a, b) with the same derivative, then $f(x) = g(x) + C$ for some constant C.

9. (a) Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \pi/2$ using three approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate and underestimate or an overestimate?

   (b) Repeat using left endpoints.

10. A car is traveling at 60 mph when the driver sees an accident 0.1 miles ahead and slams on the break. What constant deceleration is required to stop the car in time to avoid a pileup?

11. (a) Let $f(x) = (5x^3 + 2x^2 - 8x + 1)^9$. Calculate $f'(x)$. Compare this to $f(x)$, and think about how you may be able to work backwards to find $f(x)$ if you were only given $f'(x)$.

(b) Suppose $g'(x) = 6(2x^3 + 9x - 5)^5(6x^2 + 9)$. Can you find an antiderivative $g(x)$ of $g'(x)$?

(c) Suppose h and u are functions. Can you find an antiderivative of $h'(u(x))u'(x)$?